Physical Meaning of Computation

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Abstract

The long-lasting discussion on the physics of computers is reexamined. Based on the fact that the information stored in the computers has no connection to their thermodynamical properties, we derive some results disproving the former studies on the thermodynamical limit of computers and brownian-motion computers. It is also shown that the physical entropy and amount of information should be treated separately in the physical discussion of computation. The physical meaning of computation is discussed from a general viewpoint, introducing the reciprocal relationships between the fundamental properties of computers. Further, a general measure of computation and its relation to the general definition of complexity are discussed based on a phenomenological model of computation.

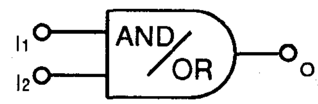

Figure 1: AND/OR gate.

I_1	I_1	O(AND/OR)
0	0	0
1	0	0/1
0	1	0/1
1	1	1

Table 1: I/O of AND/OR gate.

1 Historic background

1.1 Thermodynamical limit of computers and logical irreversibility

There have long been discussions about the fundamental limits on computation. The discussion is considered to have been invoked by Landauer [1] early in the 60's. He was the first to examine physical limits on computers, though Brillouin should be acknowledged for his important earlier work on computation. [2] Computers as physical objects must follow the laws of thermodynamics. Also, analogously to information theory, Landauer thought that the change in the physical entropy of a computer should be correspond to that of the amount of information stored in it. Landauer used these assumptions to derive thermodynamical limits of computation.

Unlike in communications, the input information is usually reduced by ordinary calculations. A simple algebraic form 1+2=3 makes this clear. There is no way of reconstructing the left-hand side of the formula from the other side. This implies that calculation may be an irreversible process.

Looking inside computers, this is clear. All modern digital computers consist of fundamental logic gates, for example AND, OR, NOT and FANOUT. Conversely, such a small set of logic gates can make up any kind of logic operation. It is clear in Figure 1 and Table 1 that the former two logic elements destroy part of the information.

They have two ports for the input but a single port for the output, and each port carries binary signals. The amounts of information in the input and the output I_i, I_o are expressed as

$$I_i = \ln 2^2, I_o = \ln 2.$$

This shows that $\ln 2$ units of information is lost during every operation of the gate. [3] Therefore logic gates, the most elemental parts of computers are irreversible. This is called "logical irreversibility."

Assuming that the gate is thermodynamically separate from the environment, its change of information should be equal to the change of physical entropy.

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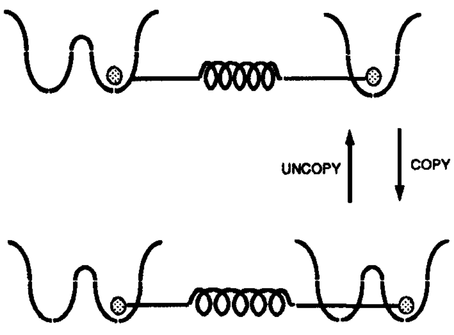

Figure 2: *Uncopy* function.

Thus, the increase in physical entropy of the gate is

$$\Delta S \equiv k_B(I_i - I_o) = k_B \ln 2,$$

where k_B is Boltzmann's constant.

Direct application of the second law of thermodynamics indicates that the gate should generate heat given by

$$\Delta Q = T\Delta S = k_B T \ln 2,$$

where T is the environment temperature.

The result leads to the statement below.
" $k_B T \ln 2$ of energy should be transferred to heat per bit of information processed"
This is called the thermodynamical limit of computers.

The quantity $k_B T \ln 2$ itself had already been known [2, 4] and is impractically small, even compared with the energy consumed by the latest gates (on the order of $\sim 10^{-9}$ [J] [11]), the discovery of an energy loss essentially due to logical operation was sensational. The discovery implied that even a purely informational entity such as logic should be subject to thermodynamics, which is a part of physics.

As shown later in this article, the thermodynamical limit turned out to be incorrect. However, it is notable for having brought suspicion on the firmly believed assumption that physics and information are always separable.

1.2 Reversible Computation and Brownian-motion Computers

Keyes and Landauer's *Uncopy* function described in Figure 2 showed that information in a memory can be erased reversibly if it has already been copied to another memory. [5] This means that information duplicated during calculations can be removed without losing energy. Following this, the irreversibility in computation was disproved by the Brownian-motion computer. [6] The state of this computer wanders around its defined configuration space reversibly, as shown in Fig. 3. It is driven only by thermal fluctuations, so the free energy is never consumed. As the trade off for reversibility, the state of the computer never be certain during the calculation. The state goes back and forth along the programmed path, or in a labyrinth which is well separated from the irrelevant area of the configuration space. Therefore, no period of time can be guaranteed to get the answer. The time can be shorter and more certain if an external force is applied to drive the calculation onward. Its motion in the configuration space is shown in Figure 4. In this case, the motion is no longer reversible and must consume some amount of energy, which need not be bounded by $k_B T \ln 2$ per processed bit. This seems to be analogous to quasi-static motion in frictional media, although the origin of the dissipation was still ascribed to the loss of information due to the decrease in the number of possible trajectories of the states as described in Figure 5.

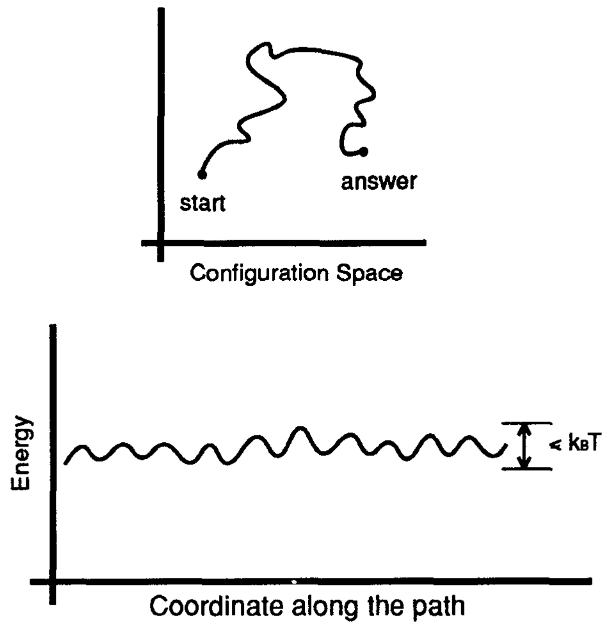

Figure 3: Above: Locus of a Brownian-motion computer in the configuration space. Below: Potential scenery along the path.

After the claim that computers do not necessarily dissipate energy, many imaginary setups were invented to demonstrate dissipationless computation, similar to perpetual motion machines of the past. [8] Some are based on quantum mechanics. [9, 10]

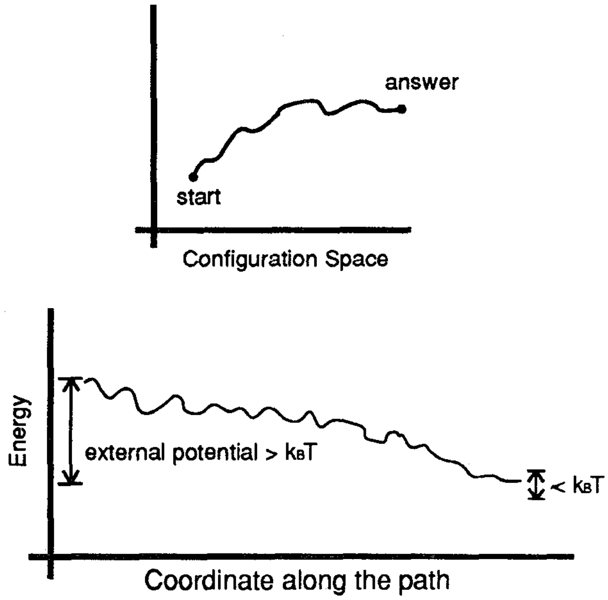

Figure 4: Above: Locus of a driven Brownian-motion computer in the configuration space. Below: Potential scenery along the path.

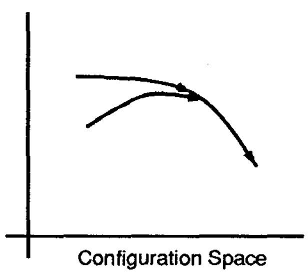

Figure 5: Merging of trajectories in a Brownian-motion computer.

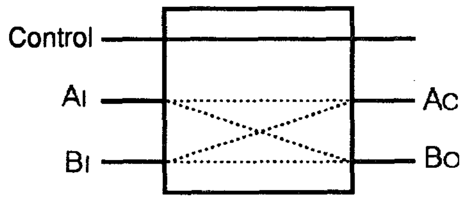

Figure 6: Fredkin gate.

A_I	B_I	control	A_O	B_O
1	1	1/0	1	1
1	0	1	0	1
1	0	0	1	0
0	1	1	1	0
0	1	0	0	1
0	0	1/0	0	0

Table 2: I/O relationships of Fredkin gate.

1.3 Reversible Logic

Although the Brownian-motion computer demonstrated reversible computation, nothing was mentioned for logic elements until 1982. Fredkin and Toffoli claimed that it is possible to implement all the ordinary logic gates without any loss of information by a simple circuit named *Fredkin gate*, which is illustrated in Figure 6. [7] Its function is obvious from the Input/Output relationships shown in Table 2. For example, by setting A_I as 0 and considering both B_I and Control as the inputs, AND will be put out at A_O.

The idea of logical irreversibility is disproved by this gate. However, it does not necessarily disprove the thermodynamical limit. The *Fredkin gate* revealed that the elemantal logic gates can be separated into the essential logic function and erasure of useless information. Although logic itself does not necessarily need dissipation, if the conventional logics gates are to be emulated, $k_B T \ln 2$ of energy is still required to dispose of garbage information.

The real limit of computation cannot be discussed without considering the garbage treatment. In order to proceed, we need to consider computation from a more general viewpoint.

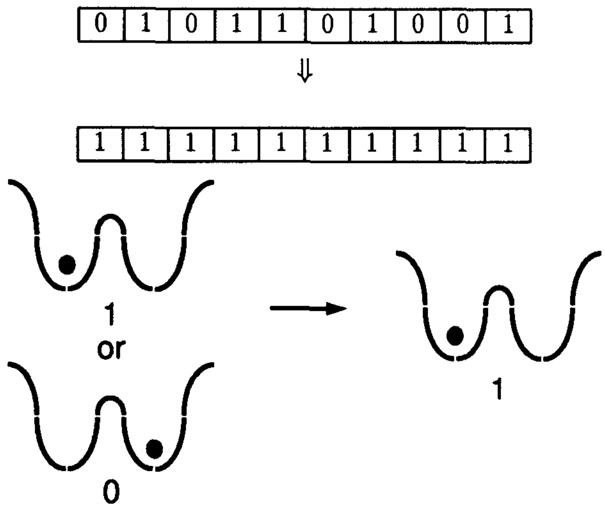

Figure 7: Above: *Restore to One* action. Below: Its Elemental Process.

2 Physical Meaning of Information

2.1 Physical and Informational Entropy

In deriving the thermodynamical limit, Landauer regarded that the *Restore to One* action reduces the physical entropy. [1] *Restore to One* changes all bits to the same state *One*, whichever state they were in before, as shown in Figure 7. In reality, the physical entropy does not change because there is no thermodynamical difference between *Zero* and *One*. Each of them is definite and has no statistical factors. Also, the physical entropy of the states *Zero* and *One* can be the same by physical symmetry. Of course, it is possible to make asymmetric memory states, in which an amount of energy should be dissipated corresponding to the entropy difference between the two memory states. However, this dissipation is trivial in the discussion here. Therefore, it is possible to state that all combinations of ensemble states have the same entropy.

Landauer thought that the *all in One* ensemble would have a lower entropy than a random one, because the spatial ensemble of memories can be converted into a temporal one. Although convertibility may hold for ergodicity, the randomness of the bits never influences the thermodynamical values. This is because even temporal changes of the states do not affect the physical entropy, as far as each state is definite during the sequence.

It seems that different kinds of entropy have been confused. A definite state has non-zero information only for somebody who does not know about it, but it has no information for those who know what the state is. Such a non-objective amount must not affect the physical state; it has a purely informational meaning. This amount of information should be called the *informational entropy*.

The physical entropy increases only when the state becomes indefinite. An indefinite state is uncertain to all, because it fluctuates. The fluctuation differs from temporal bit ensembles because the change is driven by microscopic dynamical variables which cannot be controlled. The heat generation in *all in One* was confused with the process of changing an indefinite state to *One*. During this, an energy of $k_B T \ln 2$ is dissipated. However, the energy does not correspond to heat-generation but should be regarded as to stop keeping initial refrigeration in preparing the computer. Such a thermalization process will break the system and never help it to work.

The definition of the physical entropy is not perfectly objective but depends on where we define the border between kinetics and thermodynamics. Fortunately, it is set automatically when the observable scope is defined. In the case of computers, this concerns the preparation of the information channel but has no relation to the information contained at any time. In other words, it can be stated that physical entropy only relates to the stability of the states and thus to the accuracy of the results.

In most cases, informational and physical entropy can be regarded as the same. This is the guarantee that communication and computation can be performed by physical entities. However, they should not be confused when the relationship between physics and computation is considered.

2.2 Energy cost of quasi-reversible computation

Let us reexamine the reversibile *Uncopy*, which has been the basis of the discussion on reversible computation.

In the informational sense, there is no reason to impose a quasi-static limit on the reversible *Uncopy*. It is obvious that copying the states inside computers produces no additional information, and neither does *Uncopy*, which is the converse. *Uncopy* does not destroy any original input information. It only switches between the memory states.

The need for a quasi-static operation for the dissipationless *Uncopy* is derived from the switching pro-

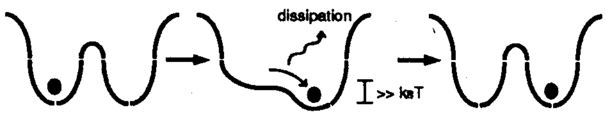

Figure 8: Conventional lossy switching.

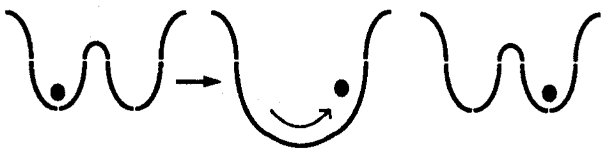

Figure 9: Lossless switching.

cess. Dissipation plays an essential role in the conventional switching process as illustrated in Figure 8.

It seems that the less energy lost, the slower it works, though this idea cannot be extrapolated to the quasi-static limit as shown later.

The need for dissipation is proven not to be general by the lossless switching action shown in Figure 9. It can switch without energy loss in a finite period of time. However, in classical setups, dissipation always exists regardless of whether it is necessary. Thus, the ideal motion in Figure 9 must actually suffer from dissipation. Before going further, another factor, the accuracy of the motion, should be taken into account. On a microscopic view, the energy of the states has the distribution shown schematically in Figure 10. Due to the dissipation and thus the agitation from uncontrollable variables, the probability distribution of the memory specifier gets broader after a swing. Dissipation-free operation is possible but may fail if the state energy falls lower than the original potential minima. Accuracy can be improved by setting the discrimination level lower, but this inevitably introduces dissipation. The system should be charged when the potential reforms into the original bistable

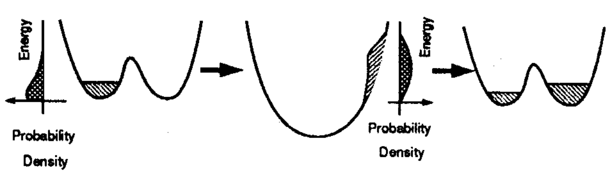

Figure 10: Microscopic view of lossless switching.

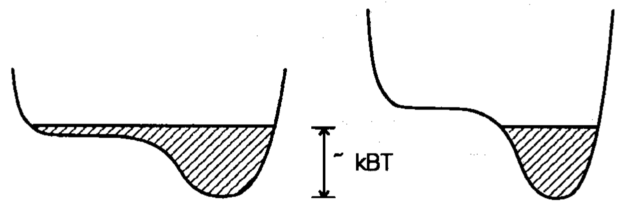

Figure 11: Microscopic view of lossy switching.

shape. It should be emphasized that the origin of error is not the free kinetic energy itself but the broadening of its variance due to the thermal environment. It can be said that dissipation essentially concerns accuracy. Dissipation relates to the speed only when remaining free energy reduces the accuracy of the motion.

If we use an essentially microscopic mechanical setup disconnected to the thermal environment, it is possible to break the thermodynamic limit on dissipation and accuracy. However, it is impossible to avoid quantum mechanical limits imposed by uncertainty relations. In particular, time-energy uncertainty forbids a definite energy level in a finite time period. This restricts the maximum speed for a given system energy, which is not dissipated, though.

The reciprocal relation between dissipation and accuracy is so general that it is also found in the conventional lossy switching scheme. In a microscopic view as in Figure 11, the potential difference cannot be smaller than $k_B T$. Otherwise, the probability distribution would spread over the entire region. Similar to the lossless case, the deeper the well, the more accurate the result. Thus, the infinitely small loss in quasi-static operation is misleading. At the limit, the accuracy goes to zero and can be improved only by a sacrifice of energy. The conventional method never has dissipation less than $k_B T$ for an acceptable accuracy. This is the reason why the simple analogy to quasi-static motion in frictional media is not precise enough for this discussion.

Since there is no need for energy dissipation in switching other than for accuracy, the idea that the reversible *Uncopy* procedure needs a quasi-static action is clearly untrue. A reversible *Uncopy* can be implemented in a finite time, which can be shorter if a larger error rate is permitted.

In the erroneous discussion of quasi-static motion, a small amount of dissipation resulted from the loss of information. However, the factor of $k_B T \ln 2$ did not arise in the precise discussion above. $k_B T$ would be the key factor in the reduction of the error rate, but our discussion did not use the factor $\ln 2$. In fact, the amount $k_B T \ln 2$ does not derive from switching nor *Uncopy*, but only from a purely logical requirement when a bit of information actually disappears. The description for the dissipation in the Brownian-motion computer is either negated. As described in the section 1.2, Brownian-motion computer becomes likely to terminate in a finite period of time when an external force is applied. The period tends to be shorter if the force increased.

Although, physical entropy of the computer decreases during calculations, origin of the energy dissipation cannot be attributed to the decrease of the degrees of freedom in trajectory of the states as described in Figure 5. It is clear that the computer does not lose any information because its reversibility will be restored if the force is released. The description was mislead by forgetting the fact that those merged paths were originally diverged from the same point at the initial stage of the calculation. Looking the whole motion of the computer, Figure 5 is completed as Figure 12. The number of possible paths is reduced, so the physical entropy should be reduced. However, the original information can be reconstructed by tracing back any path. Further, the paths often merge even without an external force. The real reason for the dissipation is the increase in the physical entropy of a part of the system not utilized to carry information. The physics of this part should be closely related with the part containing the information to drive the calculation, but should not interfere with it. The feasibility of a Brownian-motion computer depends upon whether such a connected system is allowed or not.

A mechanical model of the *Fredkin gate* is enough to show that finite-speed operation is possible and free from dissipation. [7] Dissipation and accuracy are related in the same manner as in the result for *Uncopy* and simple switching. Dissipation is necessary to stabilize the ballistic paths, which accumulate errors at reflections.

There is another limit because the operation speed is given by the kinetic energy of the balls. The limit is simply due to the time-energy uncertainty for the ballistic logic gate.

2.3 Energy cost for logic

The analysis of irreversible logic gates can be simplified by separating the function into a reversible action and an information erasure. Comparing the *Fredkin gate* with the AND/OR function described in Figure 1, both garbage and the input information disappear.

Let us again assume bistable well memories to analyze the energy cost of erasing the garbage. It is convenient to assume a memory state degenerated in energy in order to analyze the dissipation due to information loss. Although garbage information may be erased by *Uncopy* using input and output information, the input itself cannot be erased from the physical state of the gate by either *Uncopy* or *Restore to One*. It is done only by transferring physical entropy, thus we must analyze the thermodynamics. The essence of the logical irreversibility lies here.

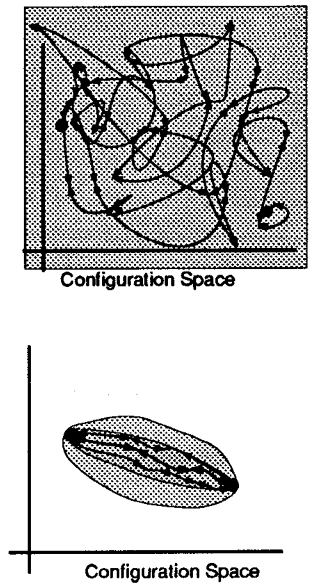

Figure 12: Trajectories in a Brownian-motion computer: Above, Free motion, Below, Driven by External force.

However, the associated heat flow does not lead the thermodynamical limit. In order to destroy information in a physical system, a definite state should get into an uncertain state. This causes entropy going out, not coming in. As a result, the gate should absorb heat from the environment. This goes against the well-known belief, but not against thermodynamics, because the gate should be considered to be colder when it contains information. [2] It does not necessarily mean costless erasure is possible. As described in the section 2.3, such a process destroys information channel. It is not suitable as a device for computers.

The story cannot complete within conventional physics as thermodynamics nor statistical physics but it is unavoidably required to consider the total procedure of computation.

3 Physical Meaning of Computation

Even after this precise discussion on information discarding and the costs of reversible and irreversible logic, the original question is unanswered, namely, "is computation possible with no cost?" It is difficult to answer the question because it is not scientific. Neither computation nor cost is defined physically. We know enough about elemental logic functions, but we do not know how much computation can be done by each of them. We have been focused only on energy dissipation, but there are other factors like speed and accuracy. In this section we will try to consider the cost of the computation itself from the physical standpoint.

3.1 Fundamental Costs of Computation

Using *Fredkin gates*, one can construct a computer that works with no loss of information, and thus no loss of energy (assuming a perfect physical setup).

However, such a computer is unrealistic. It turns out bits of garbage information at every step. It seems that the cost of energy is simply converted to the cost of the memory space required. Accordingly, the number of computation steps should be limited by the size of the computer, otherwise it will be flooded with garbage bits. Even if it works for a single trial, it must fail soon due to garbage accumulation. Such a limited machine cannot be regarded as a computer. In order to make an accurate discussion, we should make a proper model of the processing. Phenomenological observation of computers shows that the fundamental performance factors are computation speed, system size, accuracy, and energy consumption. The costs of computation can be defined as negative expressions of these performance factors. The important point is that the factors are usually related to each other as shown in Figure 13.

If we want to reduce the memory space required in the dissipationless computer made of *Fredkin gates*, energy may be dissipated according to the saved space. As described in section 2.3, accuracy can be increased if we increase dissipation. However, these statements are not true in general but are valid only when the size and the speed of the computer are assumed to be constant. Otherwise, redundancy due to larger size can improve accuracy by much more. A slower speed also help in the quantum mechanical setup as in section 2.2 or if a dumping mechanism is utilized. Moreover, speed and space are directly related by means of

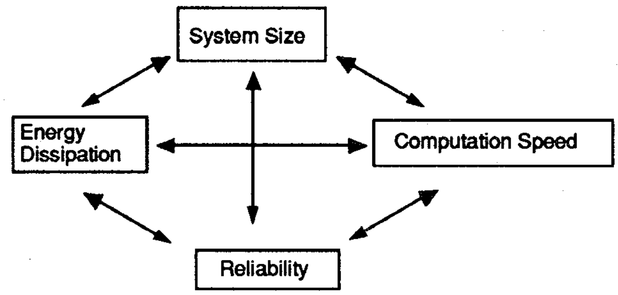

Figure 13: **Reciprocal relations between the cost of computation.**

the traveling time of the information carriers.

These relationships show that the cost of computation has been discussed on incomplete bases. If we add the lacking conditions, the question can be given scientifically as follows.

"Can we compute a task without energy with enough accuracy in a finite time under the condition below?"

The condition is one of the following:

1 The computer has a large enough working space, and is programmed and ready to calculate. Only one job is considered,

2 The computer has a large enough working space, and is used repeatedly for various calculations,

3 The same as 2 except that the working space is finite.

The question with condition 1 is trivial. No energy cost is required even under 2, because all other cost is compensated for by the space. It is necessary to use condition 3 to reveal the reciprocal relations in Figure 13. No serious discussion has been based on condition 3, which corresponds to considering the garbage problem in computers using reversible logic.

Even though we have discussed the cost of information erasure, it is not incomplete as the reusability was not considered. However, we would like to stop here for the limited space. The problems of reusability and input information erasure will be discussed elswhere.

3.2 Physical Model and General Measure for Computation

We have no physical definition of computation.

The *Fredkin gate* is reversible. Thus, no loss of entropy is required during the operation. However, this does not apply to computation, which cannot be

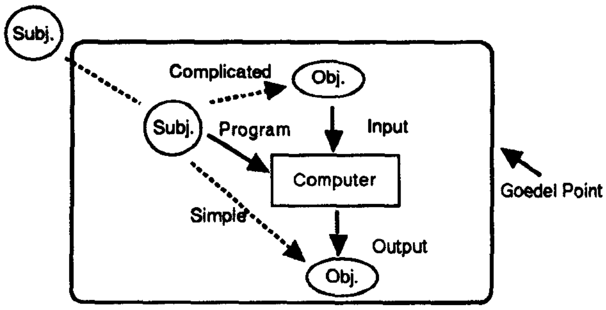

Figure 14: **Physical model of computation.**

understood as a mere assembly of such primitive functions.

This fatal problem arose from the fact that we do not have any measure for the amount of computation which, should correspond to Shannon's entropy in communications. Therefore, it has been impossible to talk about computation quantitatively.

In the simplest sense, computation can be defined as the process of making information more valuable. Thus, the physical model of computation should contain the operator of the computer as an evaluator of the result. The observable scope includes subjective information. Although this might depart a little from the conventional methods of natural science, it is still scientific as far as it stays within the Gödel point which separates tractable and intractable problems. A model which gives a good understanding of computational procedures is as shown in Figure 14

Finding a measure of computation will be an extension of Shannon's work, because communication is included in computation as the 0 computing limit.

Kolmogorov's amount of information or algorithmic information is one possibility, and is defined by the minimum program length to produce an object. [12] The theory has rarely been applied to physics, because it can only be defined for a particular computer, if the value is to be finite.

However, this may be suitable as a measure of computation in our model. Because the subjective information can be correspond to a virtual computer which mathematicians have been strugled to erase in the theory.

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$$(P_1 + P_2 - P_1P_2)\ln(P_1 + P_2 - P_1P_2) - P_1 \ln P_1 - P_2 \ln P_2 - P_1(1-P_2)\ln(1-P_2) - P_2(1-P_1)\ln(1-P_1).$$

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