Paradoxes and the Distribution of Probabilities

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Abstract

Within a classical logical environment, the occurrence of ¬A while A is the case, threatens the functioning of the whole system. In this paper, I argue that Tarski's solution to paradoxes, meta-levels, and Gödel's incompleteness theorem can be brought together as constitutive elements of a generative system. In this general system, a new level of complexity is created each time the existing levels cannot deal with a paradox on the highest level of the system. In this context, the increasing complexity of the world appears as a solution to paradoxality.

Furthermore, I suggest that the jump from one level of complexity to the next can be understood as the reassignment of probabilities. Low-probability events causing a paradox will be assigned a higher probability on the next level. As a result, the organisation of that higher level will pay much more attention to the potential trouble.

1. A generative system

Let us consider a system (eg. a world or a language) able to refer to itself and thus prone to paradoxes. In such a system, Tarski's notion of meta-levels can be used to construct a new, higher level that enables us to neutralise the paradox. This solution is similar to putting *This sentence* in quotation marks to avoid the paradox in *This sentence is false*. To replace *This sentence* by "*This sentence*" is to eliminate the self-reference of *This sentence* and to replace it by a reference made from the level of the quotation marks.

The new level of complexity is constructed based on a selection of elements pertaining to the level on which the paradox occurred. The new level, however, leads to some extent a life of its own: the new, true statements on that level do not regard individual elements involved in the paradox. As such, it is not necessary to use quotation marks to indicate the new level: The increased complexity of the relations among the elements that constitute the new level provides an admittedly fuzzy, but nevertheless sufficient indication.

Gödel's incompleteness theorem implies that the now again consistent system will contain true statements that cannot be proven by the system. When a system creates a next higher level to regain the consistency that it lost when one of its levels ran into paradoxes, then from Gödel's work it follows that the newly created next higher level will contain a true statement that cannot be derived from the lower levels.

The evolution of the world can be described in terms of such a generative system. Life, for instance, is not about the behavior of individual carbonmolecules, notwithstanding the facts that life would not be what it is without these molecules and that we cannot tamper with those molecules without menacing the conditions for our existence. That is precisely the point to be made: While securing consistency or coherence with the evolutionary roots, the newly created level allows for novelty beyond what could possibly have been accomplished at the level of those roots. As a result, the levels of reality indeed cannot be reduced to one another without losing the consistency gained when new levels of complexity were introduced.

It appears to me that a gap—a jump—between levels is actually a good strategy: It is the strategy used by the world itself when it creates new levels of complexity on which it can forget the paradoxes on the lower levels. Those gaps are essential to the consistency of the world. The jumps between object-level and meta-level are real indeed.

2. The distribution of probabilities

Whereas the results and advantages of level jumps are quite obvious, the process itself is not. I suggest that a jump from one level to another can be understood as the reassignment of probabilities.

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Novelty can be expressed in terms of probability: This paper, for instance, as an event in space-time, has an extremely low probability on the level of chemistry, a somewhat higher probability on the level of biology, and an even higher chance to occur once we reach the cultural level.

More reality is created every time a level jump takes place to accommodate some low-probability event that threatens to get the system stuck in a bottleneck by causing a fundamental paradox. Every level jump, in that sense, is a "guess" about how the machine can be kept running smoothly. Since the working of the machine is based on positive as well as negative feedback, small changes—unexpected or low-probability events—can lead to major trouble. At some point, the information in the system is no longer self-healing, the problem must be analyzed, and a new code-protection must be installed.

Paradoxes are the bottlenecks of the complexity that can be dealt with on the level of reality on which they occur. These limits are inherent conditions of any generative system and can as such not be avoided. At the bottleneck of the quantum-mechanical level, light behaves as both a wave and a particle. Going through the bottleneck solves that paradox by adding an arrow of time to reality: Light can be observed as a wave and as a particle, but not at the same time. In the statistical pattern created by the process of going through the bottleneck—almost literally in the case of the double-slit experiment—time gains a preferred direction, which appears to be absent in the interaction on the quantum level. In other words, the probability of one direction has been increased while the probability of the other has been decreased.

From that same perspective, life appears as a radically new way to protect information against the disintegration of physical and chemical structures. Along those same lines of thought, our mind might be the most sophisticated device which the system has come up with so far, a device capable of experimenting with the long-term (paradoxical) effects of the solutions that it is going to implement.

3. Increased texture

What emerges here is a picture of the world as a system of layers of increased complexity, whereby each layer originates as a form of partial self-reference of the previous layers. It must be assumed that the new layers do not add to the amount of matter/energy in the universe, so that it may be more appropriate to speak of the increased texture, or the increased density, of the world.

In terms of computation, this increased density can be seen as a simultaneous increase of two apparently opposite poles. Traditionally, black and white—the prototypical opposites—are represented as extremes of a continuum. (Fig.1) However, from an alternative point of view, they can be seen as intersecting axes of a field of possible shades of gray. Within such a field, black and white can simultaneously increase in terms of their

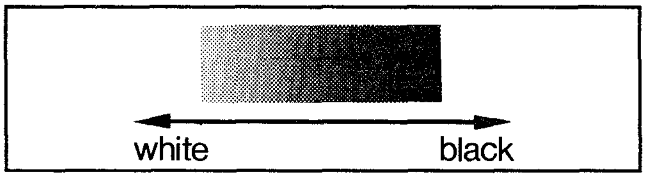

Figure 1. The increase and decrease of black and white within a linear framework.

presence in the field. If we randomly distribute black inkblots on a piece of paper, we end up with an image that moves from white over grayish to black as more blots are added. When we organize blots into a lattice, we end up with an image that is simultaneously more white and more black. (Fig.2.)

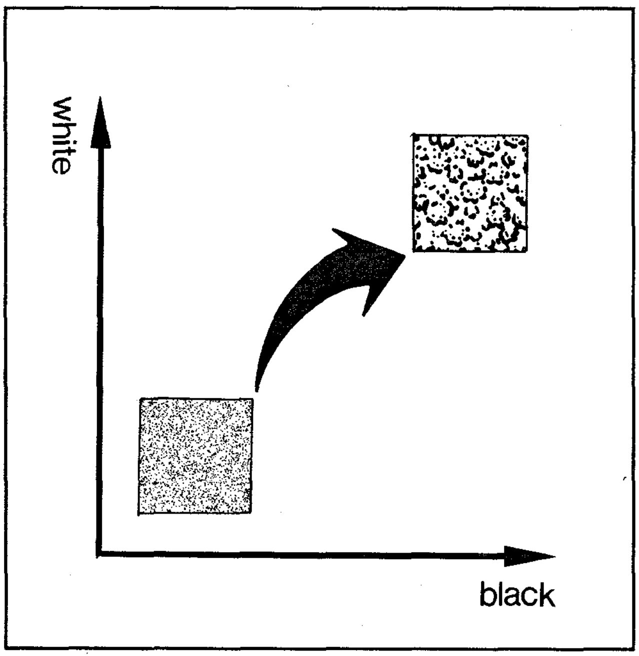

Figure 2. The simultaneous increase of black and white can be thought of as an increased order, here visualized as increased texture.

Order and disorder to some extent have to be independent from one another, because they can increase at the same time. Order and disorder—entropy and constraints—can be understood as two fundamental parameters operating in space-time. We can represent them in a 3-dimensional model, in which entropy is represented on the y-axis, constraints appear on the x-axis, and the combined dimensions of space-time are shown on the z-axis. The relation between entropy and constraints can be thought of as similar to the relation between black and white in figure 2. When entropy and constraints increase simultaneously, the order of the system increases. (Fig.3.)

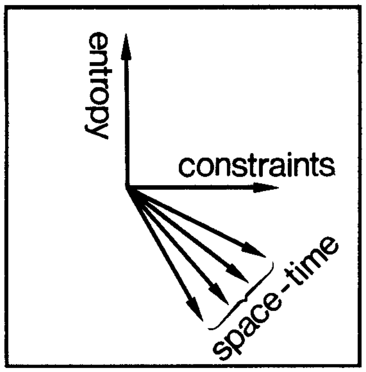

Figure 3. A world can be defined by time-space, entropy, and constraints.

4. Rearranging probabilities

To be meaningful, an event must have a probability greater dan 0 and smaller than 1. This probability can be described as the interaction between a range, or distribution, of entropic probabilities and a range of constraining or "texturizing" probabilities. Together, these probabilities define a field of possible events. The interaction between them through space-time creates an interference pattern that functions as a strange attractor, organizing the events at that level of complexity. (Fig.4.)

Moving from one level of complexity to another literally means rearranging or reconfiguring the probabilities that defined or organized the previous level. A paradox is not as much a matter of black and white or true or false as it is a matter of high and low probabilities.

Levels of complexity are defined by the structure of the interference pattern of probabilities attributed to events. Events, then, are informative as a function of their probability: an event with a low probability has more information-value than one with a high probability.

The system must, however, pay a price for the information it obtains at a given level. Low-probability events, taking place outside the strange attractor or resisting the attractor, can start forming a cluster of their own. When such a cluster becomes important enough, it may necessitate a modification of the original distribution of probabilities on the entropy axis and/or on the axis of the constraints, thereby modifying the attractor to accommodate the newly formed cluster.

In case the cluster cannot be accommodated—when, in other words, a paradoxical situation arises—the original strange attractor can fall apart or bifurcate and create a more complex pattern of interference between entropy and constraints. A new level of complexity has been introduced. That will be the case when the existing strange attractor cannot be modified without simultaneously doing away with the solution to the paradoxes on the previous level. If that were to happen, the whole system would collapse to that previous level, and the whole paradox-solving procedure would have to be started all over again.

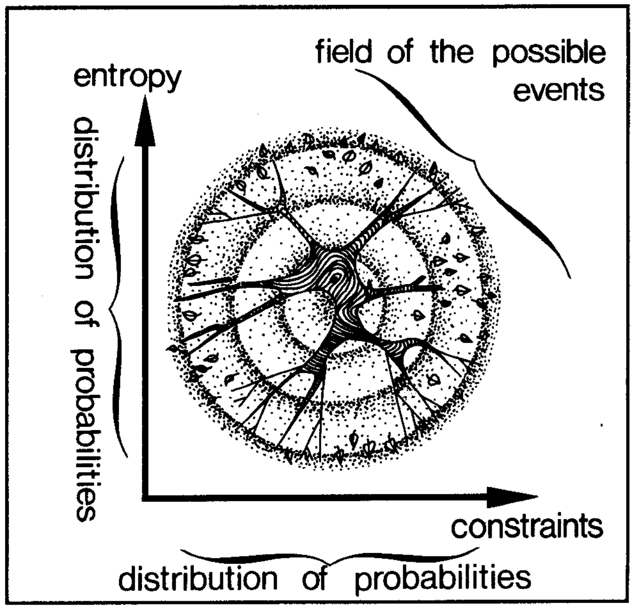

Figure 4. The interaction of entropic and constraining probabilities defines a field of possible events.

In many or even most cases, the paradoxes created by a low-probability event will not be, or cause, a real problem. When we throw a stone into a lake, everything becomes normal again after a short disturbance of the surface of the water. The interference came as a total surprise to the lake, but it never really fundamentally disturbed the lake's peaceful and smooth existence.

In other cases, however, the unexpected interference will start a chain of positive feedback, leading to a dramatic reorganization, a fundamental redistribution of the original probabilities. The emergence of life appears to be a low-probability event. But life, as the result of the redistribution of probabilities—the evolution of probabilities—may become increasingly important in its interference with the universe.