Space, Time, Logic, and Things

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Abstract

In this paper, we examine the fundamental origins of logic and show how these fundamentals are related to basic concepts of space, time, objects, and events used in both physics and computing. We attempt to show how a universe can be constructed beginning not from first principles, but from no *principles. Several possible implications for physics and mathematics are also discussed.*

1. Introduction

In the previous companion paper [1], we introduced a logic of simple digital circuits, then extended it to show some interesting behavior which might be relevant to quantum mechanics and other physical theory. In this paper, we concentrate instead on the fundamental origins of this logic and show how these fundamentals are related to some of our most cherished concepts. The logic is developed by the simplest steps, as if a universe were being constructed with the fewest assumptions possible. We then discuss some (highly speculative) interpretations and implications for both physics and mathematics.

In order to progress beyond current physical theory to the holy grail of a Theory of Everything, it appears that we need to find a deeper basis for some of our commonly-held notions such as space, time, objects, and events. Physicists and philosophers have for centuries pondered the origins of the universe as we know it, and for decades the paradoxical and largely statistical reality described by quantum theory [2]. Computer Science has also sought deeper paradigms for representing computation, while remaining perched firmly on the shoulders of Turing and von Neumann throughout its entire history.

We explore here the idea that such a new basis exists common to both fields which can best be understood as a primitive *boundary mathematics* along with its extension to and interpretation as *logic*. Ultimately, we seek to develop a theory consistent with both physical and mathematical phenomena, based not on first principles, but on *no* principles.

1.1 About language and primitives

A cautionary aside: It is quite difficult to speak about such fundamental matters in ordinary language. Many statements in this paper may strike the reader as terribly informal and inexact (or even contradictory), when in fact they are attempts to be quite precise in describing things and situations for which we do not have common words. While the boundary mathematics discussed here is itself ultimately formal, it is not much help in getting started since we begin beneath traditional mathematics, and there is nothing more primitive in terms of which to speak.

We first have to speak *about* the mathematics before we can use it. In contrast, most conventional mathematics begins with (somewhat higher-level) concepts which are relatively easy to state and agree upon ("object", "membership", "equivalence", "choice", "truth value", "proposition") since they are related to our everyday experience. Consider the opening sentences of Quine [3]:

> "... Sets are classes. The notion of class is so fundamental to thought that we cannot hope to define it in more fundamental terms."

We respectfully disagree, and will attempt to begin with no assumptions or agreement, literally with nothing at all.

1.2 Computer Science and Physics

Suppose you were a god and were attempting to build a universe. What primitives would you use, and how would you structure your universe?

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In a sense, this is what computer programmers do all the time. Beginning with certain (built-in hardwired) primitives such as bits and bytes, arithmetic functions, addressable memory, etc., they construct complete "universes" which model complex systems or phenomena or behaviors. Inside a computer, a configuration of zeros and ones can be made to invoke almost any behavior desired, at the push of a few keys. Computer hardware designers seem to have even more freedom, since they begin with even simpler primitives of logic gates, registers, and clocks.

If programmers and engineers build universes, this could be thought of as complementary to physicists, who study the particular universe we seem to inhabit -- the so-called "real world". Perhaps these two universes will eventually turn out to be the same -- or at least intimately related.

It appears at first glance that the computer designer's universe must be entirely classical, since his primitives are based in classical logic and carefully quantized time intervals. In fact, computers would be entirely classical if they worked 100% reliably. But real digital computer circuits are only analog approximations to Boolean logic, and can (and do!) fail due to thermal and environmental noise, quantum random effects, synchronizer metastability, and other causes -- hopefully at infrequent intervals.

But this may not be the entire story, as we shall see. It may be that our existing computer hardware limits us to classical behavior, but that computers built on quantum mechanical principles may have more a interesting range of behavior, perhaps even including all the phenomena of the physical universe as a whole.

In one sense, we are attempting here to build a physical universe as a computer scientist or engineer might, from the ground up using the simplest primitives possible -- primitives considerably simpler than those used by hardware and software designers *or* physicists today.

For the most part, the progress of physical science has been from the top down. That is, experimental results (observations) are answered by theory (models) which encompasses ("explains") the observations. The theory also makes additional predictions which can be tested by further experiment, and so on. This time-honored and effective approach continually extends what we already know and puts new substrate beneath it.

In contrast, here we attempt to jump to the end (or beginning) of the story, to find the most fundamental basis possible (for both physics and computing) and to build a universe, working back toward existing theory and phenomena.

We are not constrained here by existing physical theory or phenomena, but are inspired by salient characteristics of them. Also inspirational are some modern paradigms for representing computations, especially object-oriented languages [4] and logic languages [5], as well as the whole idea of digital design of complex systems. Ultimately, of course, the utility of this approach to physics will rest on its ability to connect with and become consonant with modern physical theory and to predict behavior of the real world.

2. Things and Space

The key concepts of our mathematics were given by Spencer-Brown in his book Laws of Form [6]. He begins with a *space* and the idea of a distinction or *boundary* in the space. In the previous paper [1], we passed quickly through the first few steps of this mathematics in order to develop a *boundary logic*, which was then extended to handle self-referent, looped logic expressions and circuits. This time, we will examine the first two distinctions more closely.

2.1 The Void and the observer

We begin with nothing -- the Void. The Void is not a thing, but the absence of any thing. It is not physical space like the vacuum, nor a mathematical space of any kind. It has no properties, and is what we indicate by not speaking. It is nothing and everything.

We need to talk about the Void in order to say what the first distinction divides. As we shall see, before the first distinction the Void is the position of the observer [1].

2.2 A first boundary

A *boundary* draws a *distinction*, and is the minimum action which makes a difference, the smallest step one can take away from the space in which it is drawn. In fact, this is the very definition of a boundary, not something dependent on any properties of a boundary nor of the space in which it is drawn, since neither of them have any properties at all. The boundary cleaves the space and thus creates the first object or thing.

For example, a circle in a plane draws a distinction (Figure 1). N.B.: Drawing in the plane is a representation, not the thing itself. Care must be taken to avoid making inferences which depend on the particular topology of any representation. The distinction being referred to has no dimension, size, shape, position, etc.

[1] By using the term "observer" here we do not intend to invoke the idea of a measurement in the quantum mechanical sense. One could substitute the word "god" or simply "you".

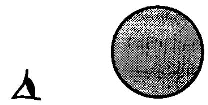

Figure 1. The observer and the first distinction.

A boundary can be seen as standing for a thing (object) as well as an injunction to cross -- the most primitive action. Thing and action, object and event are undifferentiated at this extremely primitive level. By drawing the distinction, we indicate the inside, that is, the side we are not on. The first distinction separates the observer from the rest of the universe ("creating" it!) and allows him (you) to observe it. [1]

It might be (and always is) asked at this point "Where does the first distinction come from? Who creates it?" Spencer-Brown answers eloquently [6, pg. 105]:

> "It seems hard to find an acceptable answer to the question of how or why the world conceives a desire, and discovers an ability, to see itself, and appears to suffer the process. That it does so is sometimes called the original mystery.".

There is no mystery (except to our feeble brains), no solution to seek. What can happen, does happen. There simply *cannot be* any deeper "explanation" for the origin of the universe which we see.[2]

Moreover, there is only *one* simplest thing, and necessarily so. The first thing cannot be described (in principle), it just *is*. It is for this reason that it is difficult to talk about and understand. The difficulty is not that it is too complex for us, but that it is too simple. In fact there is a very real sense in which the first thing is not "understandable" at all, since it is composed of no parts, has no attributes, depends on nothing, and is related to nothing except the space from which is distinguished.

2.3 A second boundary

Continuing to build our universe, and again taking the smallest step possible, we may draw a second boundary identical with (indistinguishable from) the first. As shown in Figure 2, there are only two places where the second boundary can be drawn: in the space outside, or inside the first boundary. (Since a boundary draws a distinction, boundaries cannot intersect.) Of course, it is an artifact of our exposition that the distinctions drawn here are ordered, first and second. What is really meant is configurations of two distinctions taken together.

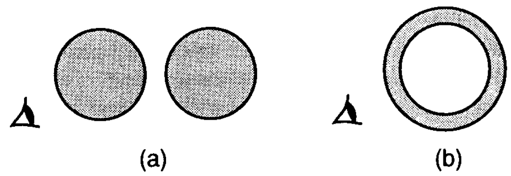

Figure 2. Two arrangements of two distinctions.

The first arrangement (Figure 2a) is symmetrical; Neither boundary has precedence or uniqueness. (There is no significance to position on the plane in our drawing.) The second arrangement (one boundary inside the other) is not symmetrical. One of the boundaries has precedence over the other and each can be uniquely identified.

As a graph, the first arrangement is analogous to nodes at the same level or a list of two items, while the second corresponds to nodes at different levels or a subgraph. List programming languages such as Lisp use structures of just these forms, plus a few "leaf" node primitives, to describe a complete range of computations and possible "worlds" [7].

As in physical theory, some states or arrangements of particles are distinguishable and some are not. At this primitive level we see that this property is not only present, but unavoidable. When our universe has only two things in it, the forms already show differences in distinguishability. Further, arrangement 2a has depth one (there is only one boundary to cross before crossing back), whereas 2b has depth two (cross twice to get to the deepest space). These properties are somewhat suggestive of the properties of fundamental particles, such as the differences between bosons and fermions, for example.

Note that it is precisely the position of the observer (outside the outermost boundary) which is responsible for the symmetry difference between the two configurations. If the observer were to (somehow) cross a boundary, the distinguishability would be reversed.

2.4 More boundaries

We can extend this arithmetic of boundaries to an algebra by allowing a letter or token to stand for any arrangement of boundaries. Figure 3 shows a more general boundary expression including five such tokens A through E, and a total of eight objects including the unnamed distinctions. Thus the innermost boundary distinguishes A and B together from the rest and represents a new object at the same level as C and D. (The

[1] "I distinguish, therefore I am."?

[2] It is tempting to draw parallels to the cosmological Big Bang theory and to the Creation story of Genesis here, but we will resist.

original observer/observed boundary is implicit and surrounds the boundary diagram.)

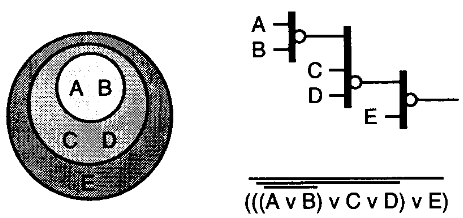

Figure 3. A boundary expression, 3 forms.

By "thing", then, we mean any object or event, physical, mathematical, or otherwise which can be definitely named or indicated, anything which is distinguished. By whom? By you who draws the distinction. The temperature is "hot" if you say it is, "not hot" otherwise. Of course, there may be different "degrees of hotness", but these degrees are different things (distinctions) than the distinction hot/not-hot.

This is somewhat like the constructivist or computable-reality view in mathematics [8], except that the observer draws the ultimate distinctions about things which are thought of as continuous, like temperature, or not commonly quantified at all, such as clouds or emotions. In this view, nothing exists which is not distinguished.

Note that we do not require here, nor do we suggest, any global or continuous dimension of Space. Space by this theory is entirely relative and in a sense local. Objects are not distinguished from partitions of Space made by boundaries we draw, so there is no notion of "matter". There is no smallest bit of space nor smallest distance in an absolute sense, yet there is always a simplest thing. A boundary can be thought of as introducing one bit of spatial resolution and two values, "Here" and "not-Here" or "There".

Furthermore, the implication here is that physical reality is entirely discrete, and that it is the observer who makes it so, and creates it in so doing. A measurement (and the attendant "collapse" of the wave function description), after all, is the drawing of a distinction. The problem seems to be that we still implicitly assume classical concepts of particles and waves as things, even while writing the quantum mechanical descriptions of them. This leads to seemingly paradoxical outcomes and correct predictions which don't "make sense". Perhaps it is not only our idea of logic which needs updating, but also our idea of object itself. By placing observer-related distinctions at the origin of our model universe, we hope to develop a much more consistent picture.

2.5 A simple interpretation: Logic

So far, the spaces and boundaries discussed are pure structure, form without any meaning or interpretation. But let us consider what the meaning of the two arrangements given above might be, based on a minimal interpretation. (See below for other possible interpretations.) In the first case (Figure 4a), each boundary serves to indicate a new space. Since the two boundaries are themselves by definition indistinguishable, they must both indicate the *same* new space. Indicating the new space twice is no different than indicating it once -- it's the same space. Therefore we say that two boundaries taken together in this arrangement are equivalent to one, as shown.

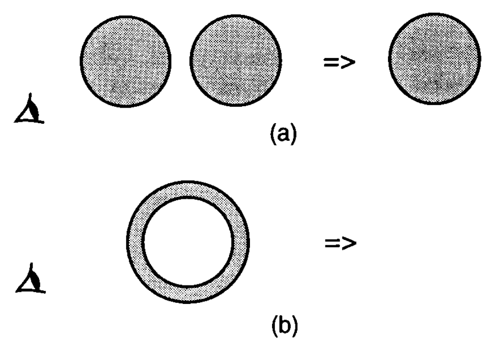

Figure 4. Two reduction rules.

In the second case (4b), the outer boundary serves to indicate a new space. The inner one then indicates a space different from the new space in which it resides. Without creating yet a third space, the inner boundary can refer only to the original outer space, the only one which is different from the one which it divides. That is, the simplest interpretation is that the innermost space is really just a continuation of the outermost. Thus two boundaries taken together in this configuration are equivalent to *no* boundaries, as shown in Figure 4b.

These two reductions can be considered as axioms of a *logic*, as Spencer-Brown does, but we have reached them by starting with nothing at all and proceeding by the simplest steps we could imagine. We prefer to think of these forms as at most theorems -- equations depending only on the existence of two distinctions. By suitable applications of these two reduction equations, any arrangement of boundaries can be reduced uniquely to either a single boundary or to no boundary [6].

Interpreted as logic, rule 4a is just disjunction or the inclusive OR function, and rule 4b can be thought of as inversion or the NOT function. Any boundary expression may be represented equivalently as a conventional logic equation, or as a digital circuit by mapping each boundary onto OR and NOT gates with inputs corresponding to the interior values. Figure 3 shows the same expression in three different forms, boundary diagram, logic equation, and circuit. We have reformulated Boolean propositional logic, with the important difference that one of the truth values has been made implicit, or cast into the Void.

3. Self-reference and Time

The boundary diagrams or circuits we have generated thus far are linear, have no loops, no re-entrance, and are entirely mathematical and timeless. They do not represent real (macroscopic) electrical circuits which have time delays and other properties.

The next step in constructing our universe is to notice the possibility of self-reference, where a circuit output loops back to become one of its inputs and an object is defined directly in terms of itself. Note that an excursion out of the original space (e.g., the drawing plane) is required. Both self reference and multiple reference are made possible by the tokens in the expression form and by the wires in the circuit form.

In our primitive world, there are two ways of making a simple loop: with an even or an odd number of inversions (crossings of a boundary) in the loop. As discussed in [1], a circuit with even inversion (consistent feedback) is an *autology* and creates a *memory*. One with odd inversion (inconsistent feedback) represents a *paradox* and creates an *oscillator*. Figure 5 shows these two simplest self-referent circuits.

3.1 Odd self-reference - Paradox

The paradox circuit (Figure 5b) defines an object X which is distinguished from itself, much like the Liar sentence "This sentence is false." We insist both on object constancy (represented by the wire), and negation (the inverting gate) for the same object. In the classical real world, we would say that two physical objects can always be distinguished by virtue of some small blemish on one of them, or by their being at two different locations in space, etc. However, at this level of simplicity, we cannot say these things, as our objects have no properties at all, only existence.

Our diagram says that X is on *neither* side (or both sides) of the boundary. The situation is over-constrained. To insist that something be "not itself", we must introduce an additional degree of freedom in the value space, and it is this dimension which leads to our concept of *Time*. Object X *can* represent one side of the boundary at one "time" and the other side at another "time". Time is precisely that degree of freedom which allows a thing to change and yet to still be itself. It is not an oversimplification to say Time *is* self-reference in Space.

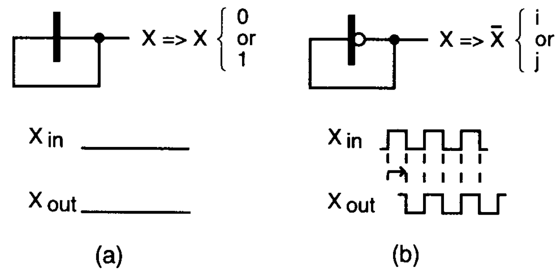

Figure 5. Two self-referent circuits.

The simplest solution is to introduce two new logic values, *imaginary values* **i** and **j**, which can be thought of as oscillations between the two original "real" values **0** and **1**.[1] The imaginary values explicitly represent the excluded middle, neither **0** nor **1**. In order to maintain consistency, we introduce a phase shift or "delay", either implicitly in the interpretation of the circuit as shown here, or explicitly in the gates themselves. It is important to emphasize that this notion of time is really *sequence*, an entirely *discrete* and *local* relation, not a time interval in the usual sense. We do not require here, nor do we suggest, any global or continuous dimension of Time. At any gate where paradox occurs, there are two values (one bit) of Time, which could be called "Now" and "not-Now" or "Then" on opposite sides of the gate.

An imaginary value could also be said to represent a *superposition* of the two real logic values. Like quantum superposition, the situation here is not a simple linear overlay of two different solutions, but an irretrievably entangled state of paradox. This may be related to correlated or entangled states in quantum mechanics. And there are two solutions, **i** and **j** (like $\pm\sqrt{-1}$), two forms of superposition, indistinguishable except by comparing or combining (interfering) with a similar waveform.

Note that the extra dimension of "time" required here is not a fully orthogonal independent dimension, just as imaginary numbers extend the reals but do not make the number field fully two-dimensional. Two imaginary logic values combine (interfere) to form a real value, just as with multiplication of imaginary numbers. See [1] for details and a discussion of the "Square Root of NOT" problem, where a real value is combined with an imaginary to yield a real result.

[1] A similar four-valued logic system for paradoxes has been developed independently by Nathaniel Hellerstein [14].

We use imaginary numbers routinely in classical physics (particularly in self-referent situations which have time-varying cyclic solutions), but they are *required* in quantum mechanics to hold the superposed, fundamentally paradoxical situations. By allowing self-reference in our circuits, imaginary logic values are necessarily generated. Analogously, when the experimenter pretends to separate himself from his experiment, paradox necessarily emerges. We have merely captured the simplest form of this in our logic.

3.2 Even self-reference - Memory

In the case of the consistent self-reference or memory element (Figure 5a), we could say that X is defined to be on *both* sides of the boundary. The situation is under-constrained. There is additional information needed to determine the meaning of the circuit. Without creating more logic values (which are unnecessary), this information must come from outside the circuit in the form of an *initial condition* specifying the state of the memory element. This additional information upon which the circuit depends we might call the Past, and again we have implied sequence -- not the usual dimension of Time but an *ordering*. The initial state of the memory must be specified so that it can represent (retain) a real logic value.

Note that a memory element whose state can be changed via external inputs (for example, a simple D flip-flop as in Figure 6) can capture and hold the state of another signal. In this circuit, the two gates on the right are cross-coupled with even feeback, thus constituting a memory element. The three gates on the left act as a switch which can connect the input signal to the memory element.

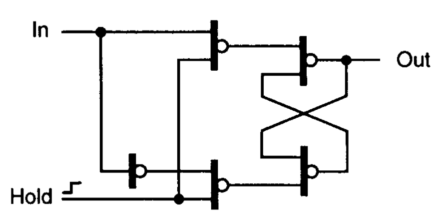

Figure 6. A measurement circuit (gated flip-flop).

When the Hold signal is **0** (false), input data flow continually to the output. The circuit is connected to or "part of" the thing being measured. When the Hold signal becomes **1** (true), the memory circuit is effectively disconnected or separated from the input, and the flip-flop remains in its most recent state.

Note that this state change is *irreversible*, since the previous state of the memory is lost whenever a new value is acquired. This behavior is suggestive of the act of measurement in quantum theory [9]. In the physical analogy, a measurement begins (and memory is erased) when a laboratory instrument is connected to (becomes dependent on) the experimental variable, and ends when the instrument is disabled or disconnected and a value is registered, either by a memory the instrument itself or by an experimenter who records it.

This model of the measurement interaction implies that the irreversible erasure occurs at the *beginning* of the measurement, and the wave function collapse occurs at the *end*, when something is recorded. During the measurement, the instrument is a part of the experiment, their wave functions are intertwined. Before and after, the two can be considered separate.

4. Implications for Mathematics

In a sense, we have reached the most fundamental mathematics, beneath the axioms normally used to begin set theory and logic. Physics is fundamentally mathematical, but there are implications for mathematics itself from this theory as well.

4.1 Set theory and numbers

The simplest interpretation of our boundary arithmetic leads to a Boolean propositional logic with one truth value made implicit. This is valuable in itself, since everything usually required in such logic (truth values, operators, parentheses) is subsumed in a single concept (the distinction) and symbol (the boundary). Any other interpretation of the boundary mathematics seems to require additional assumptions or new objects to be created as each distinction is drawn.

For example, if we allow each additional boundary in Figure 7a to define a new and distinct space (equivalently a new kind of boundary), thus denying the collapse of rule 4a, then we generate objects without order -- the cardinal (counting) numbers. If we allow each interior boundary in Figure 7b to define such a new space, thus denying the collapse according to rule 4b, then we generate ordered objects -- the ordinal (index) numbers. James [10] has explored innovative representations of numbers using arrangements of two and three distinction types.

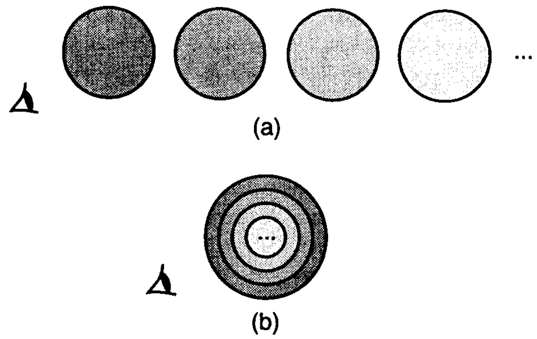

Figure 7. Interpretation for numbers and sets.

By denying both rules 4a and 4b, we get multiset elements [7] as in arrangement 7a, and set containment as in arrangement 7b. If we wish to retain rule 4a collapse only for the same type boundaries, then the result is conventional sets (without duplicated elements).

5.2 The completeness of logic

The imaginary booleans are necessary and useful for exactly the same reason that complex numbers are: They can represent the solution to any Boolean equation (arbitrarily connected circuit). The imaginary booleans are a completion to logic in the same sense that complex numbers are to the number domain. (There are higher-order booleans generated by nested looping circuits, but these can be collapsed to the imaginaries, and are beyond the scope of this paper.) It has also been shown by Turney [11] that a slightly extended form of Spencer-Brown's logic is equivalent to finite state automata.

Further, this theory gives new perspectives on a variety of paradoxes (Russell, Berry, Curry, etc.) and infinities which arise from self-reference. (See Rucker [12] for a lucid discussion of many of these, but without emphasis on their self-referent aspect.) We have explicitly denied the usual law of the excluded middle here, and shown the utility of a superimposed state which is both true and false.

For example, it is well known that some sets (e.g., the real numbers or the subsets of the natural numbers, etc.) are uncountable, by contradiction from Cantor diagonalization. Briefly, the familiar argument goes: Assume the reals *are* countable, and thus we can enumerate them in some order. But then we could construct a new number which differs from the i'th number on the list in the i'th digit. This number is different from every number on the list, and thus represents a real number which is not accounted for in our enumeration. This is a contradiction, therefore the premise (countability) must be false.

Given the logic and the discussion of paradox above, however, it seems more reasonable to say that the countability of the reals is *imaginary* (paradoxical) rather than false[1], since if a given real number is on the Cantor list, then it cannot be on the list, etc. The diagonal construction may be seen as requiring a number to disagree *with itself* in some digit, clearly a paradox.

Many such proofs by contradiction (especially those involving diagonalization) are in fact circular (self-referent), and can benefit from being seen as paradoxes with imaginary solutions. Furthermore, such constructions are unavoidable in any symbolic system powerful enough to refer to itself, as Gödel showed. For the same reasons, arguments about the non-decidability and non-computability of certain problems in complexity theory and computer science might be reinterpreted.

6. Conclusions

We have shown how to construct a universe not from first principles, but from *no* principles. Beginning from the Void, we create the first Thing (distinction) and generate objects as distinctions in Space, and events as distinctions in Time, the latter being brought about directly by self-reference in Space.

The problem at this stage is to connect this theory more fully in a formal way to modern physics. (See Kauffman [13] for a provocative discussion of Lorenz groups in relation to distinctions.) It remains to be seen whether this course of exploration can be useful in describing quantum mechanics and other physical theory of the real world. However, we have attempted to weave into the discussion some possible implications from the theory so far for physics in order to stimulate discussion.

These fundamental matters are ultimately simple (not complex), but they are decidedly not trivial (of no importance) and certainly not arbitrary (determined by whim or free choice). But the universe constructed here begins with a simplest thing (and equivalently a most-primitive action). The simplest thing is -- as it *must* be -- unique and inherently beyond description. It has no parts, no attributes, and no relationship to anything else except by composition. Since it is we who draw the first distinction (and others following it), we must in fact be "god", and this is the only way the universe could be.

[1] This idea has also been suggested by Nathaniel Hellerstein [unpublished manuscript].

Acknowledgements

Jeffrey James wrote the iLogic simulator used to explore and validate many of the circuits presented here and in the previous paper. I am also grateful to Brian Von Herzen, David Keenan, Nick Herbert, William Bricken, Dan Ingalls, Steve Saunders, and Don Charnley for many useful discussions.

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