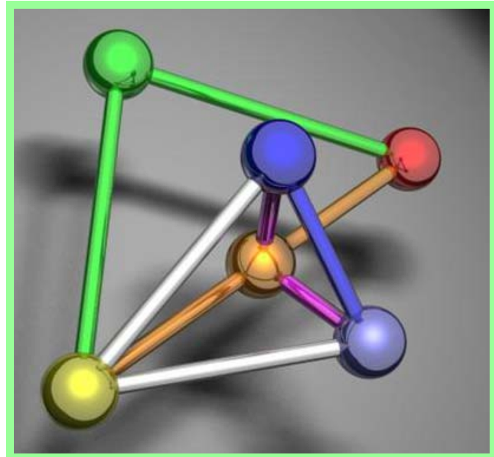

Quantum Ensemble Computing

MetroCon IEEE Presentation
Wednesday Sept 29, 2004

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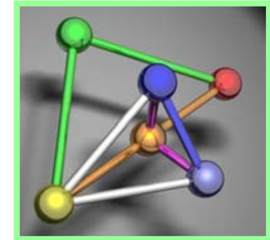
Abstract



Interest in quantum computation started growing significantly since 1994 when Peter Shor showed that quantum computers could solve some problems such as factoring, faster than classical computers. This capability is possible because quantum computers represent information state differently than classical computers. This talk will present a new set of modeling tools and concepts that can be used explore this complex yet captivating topic.

Lawrence Technologies is building a quantum computing tool set/library under Air Force contract that allows exploration of quantum computing circuits. Besides the traditional quantum operations, we have designed this library to implement quantum ensembles.

With this infrastructure in place, the topic of quantum ensembles can be expanded to include the unintuitive properties of Correlithm Objects. Correlithm Objects Theory is based on mathematical modeling of neural systems and has lead to numerous patents. I will discuss the Quantum ensemble and Correlithm Objects research, tools and results.

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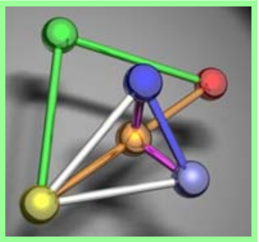
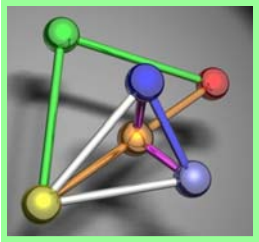
Outline of Talk



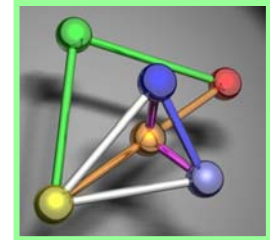
- Quantum computation basics
- Need for quantum modeling tools
- Demo of new quantum toolset
- Ensembles and Correlithm Objects
  – Standard distance and radius
  – Unit N-Cube and Hilbert spaces
  – Quantum Ensembles
- QuCOs survive measurement

Quantum Computation Basics



Topic	Classical	Quantum
Bits	Binary values 0/1	Qubits $c_0\lvert 0\rangle + c_1\lvert 1\rangle$
States	Mutually exclusive	Linearly independ.
Operators	Nand/Nor gates	Matrix Multiply
Reversibility	Toffoli/Fredkin gate	Qubits are unitary
Measurement	Deterministic	Probabilistic
Superposition	*Code division mlpx*	Mixtures of $\lvert 0\rangle \,\&\, \lvert 1\rangle$
Entanglement	*none*	Ebits $c_0\lvert 00\rangle + c_1\lvert 11\rangle$

Hilbert Space Notation



- Qubit

q0   q1

c_0 **|0>** + c_1 **|1>** c_0 **|0>** + c_1 **|1>**

not * q0
phase * q1

- Qureg

q0 | q1 | q2

$$q0 \otimes q1 \otimes q2$$

c_0**|000>**+ c_1**|001>**+ c_2**|010>**+ c_3**|011>**+ c_4**|100>**+ c_5**|101>**+ c_6**|110>**+ c_7**|111>**

- Ebit

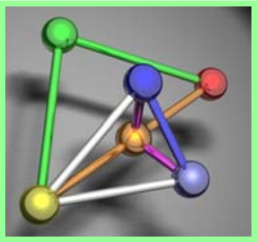
q0 | q1

$$\text{bell} * (q0 \otimes q1)$$

c_0 **|00>** + c_1 **|11>**   or c_0 **|01>** + c_1 **|10>**

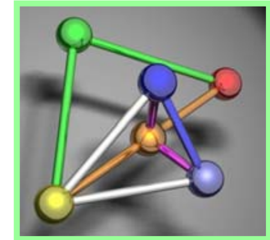
$\otimes$=tensor product

Need for QuModeling Tools



- Actual quantum computers are unavailable
- Highly mathematical paradigm shift
  - Qubits, Hilbert Space and Bra-Ket notation
  - Reversibility: unitary and idempotent operators
  - Superposition: *linearly mixing of orthogonal* states
  - Entanglement: no classical counterpart
- Facilitate learning
  - Learn notation, primitives and concepts
  - Build understanding and intuition
- Support application design
  - Model/view states not accessible in laboratory
- Next slides give examples of qubits, qregs and ebits with various operators

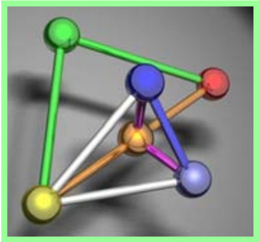
Quantum Qubit Circuits



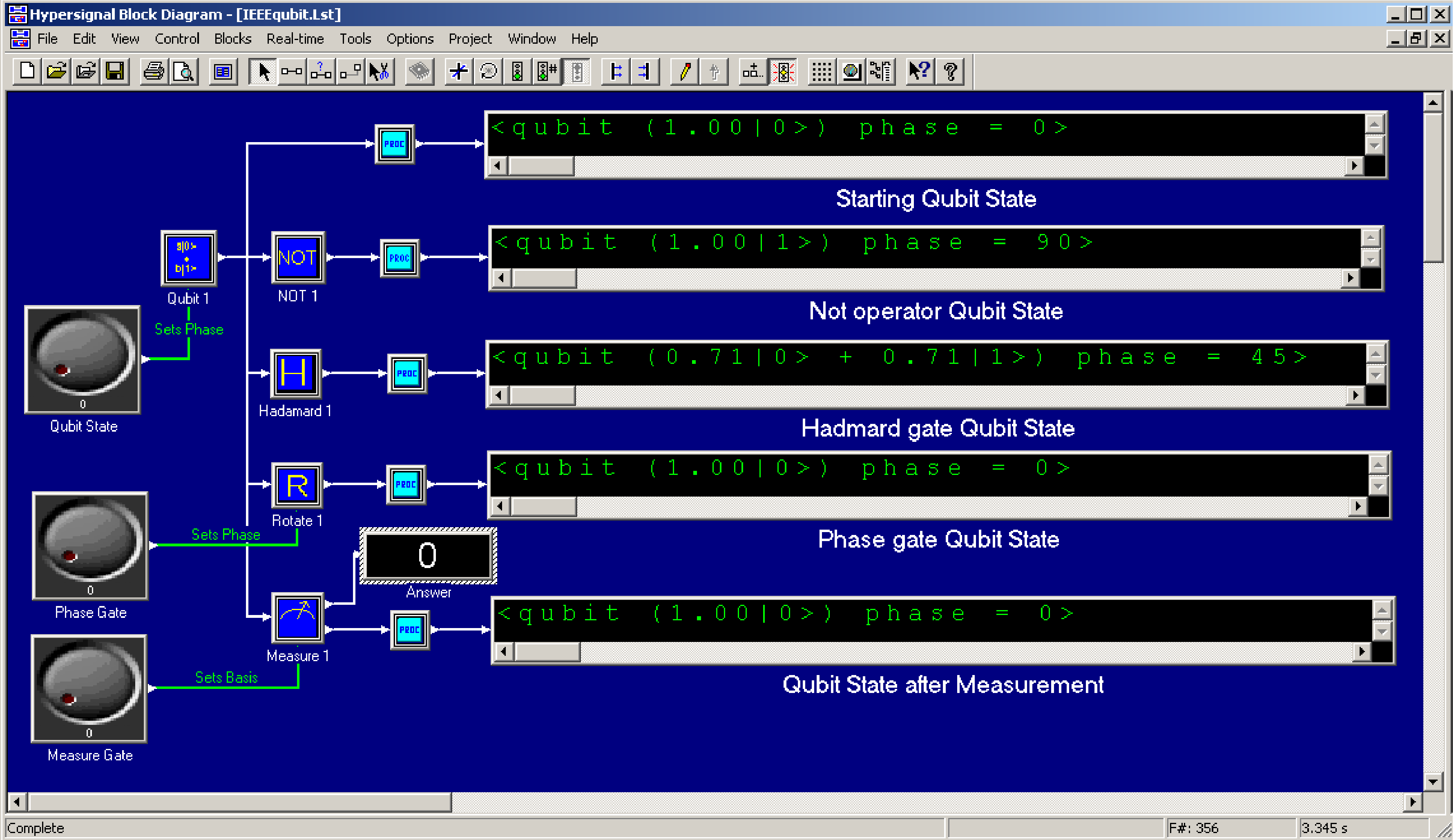
Gate	Symbolic	Matrix	Circuit
Identity	$\sigma_0 * \psi$	$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	ψ ——
Not (Pauli-X)	$\sigma_1 * \psi$	$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\psi \;-\boxed{X}-$
Shift (Pauli-Z)	$\sigma_3 * \psi$	$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\psi \;-\boxed{Z}-$
Rotate	$\theta * \psi$	$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$	$\psi \;-\boxed{\theta}-$
Hadamard	$H * \psi$	$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$\psi \;-\boxed{H}-$

$$\begin{bmatrix} |0\rangle & |1\rangle \end{bmatrix}$$

Quantum Toolset Demo



Qubit Operators: not, Hadamard, rotate & measure gates



Our library in Block Diagram tool by Hyperception

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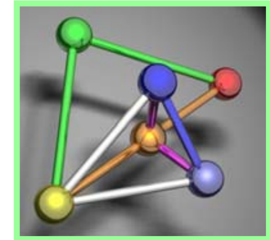
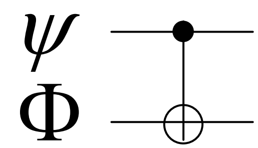
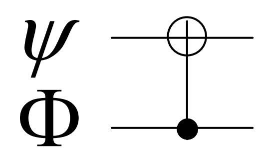
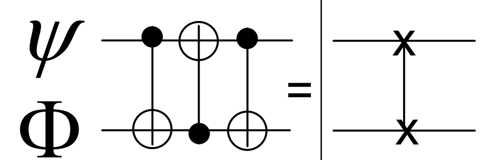
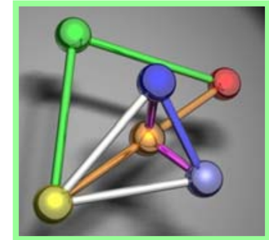
Quantum Qureg Circuits



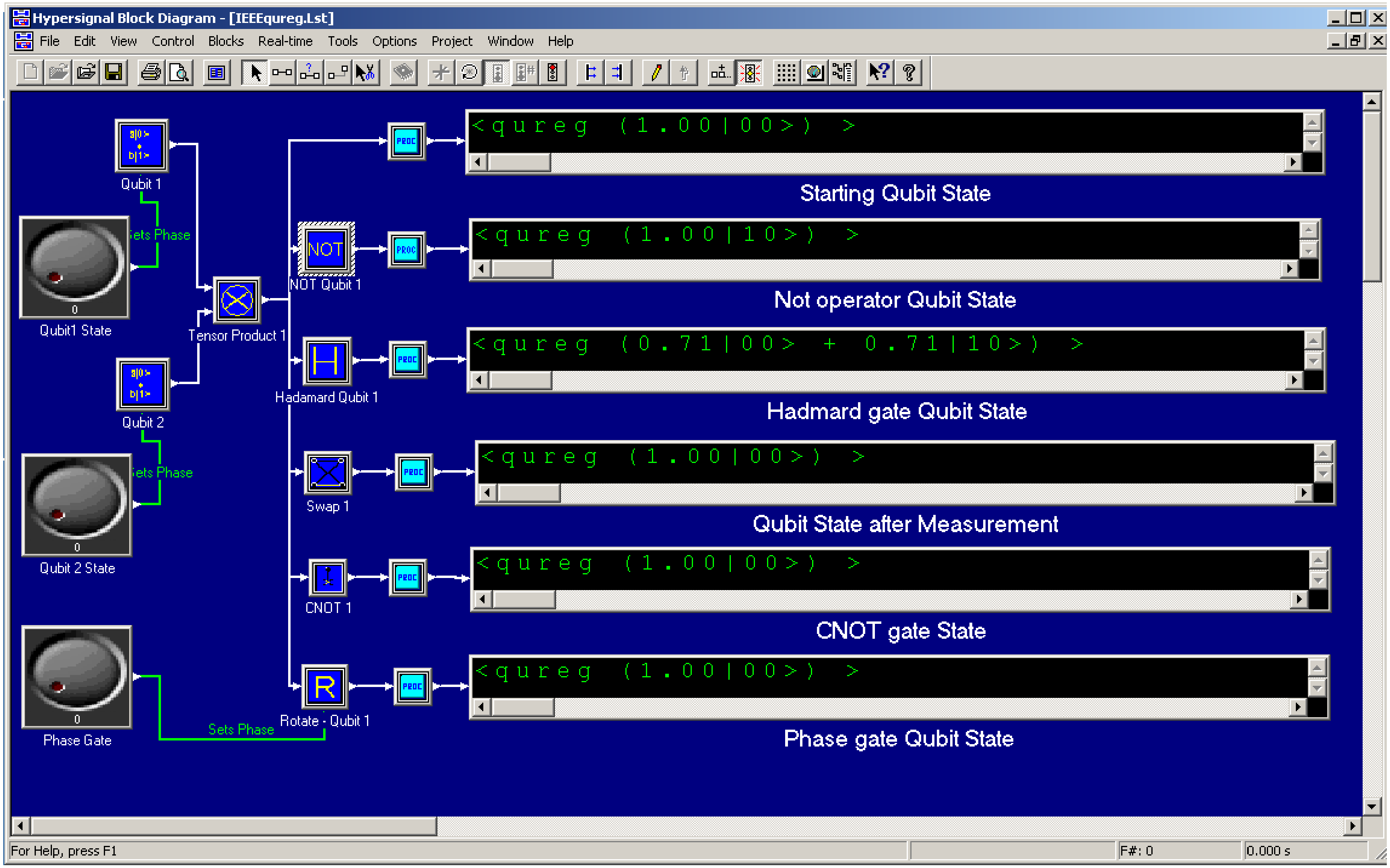
Gate	Symbolic	Matrix	Circuit
Cnot = Control-not	$cnot * \psi$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	
Cnot2	$cnot2 * \psi$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	
swap= cnot*cnot2*cnot	$swap * \psi$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	

$$\begin{bmatrix} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{bmatrix}$$

Quantum Registers Demo



Qureg Operators: tensor product, CNOT, SWAP & qu-ops



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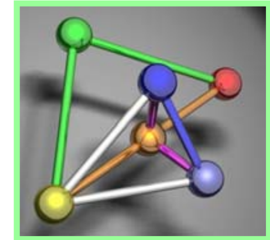
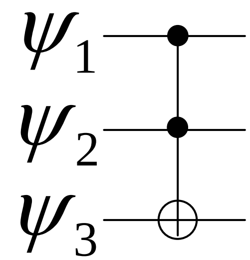
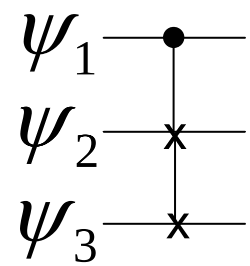
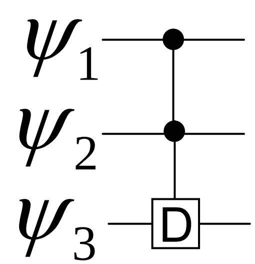
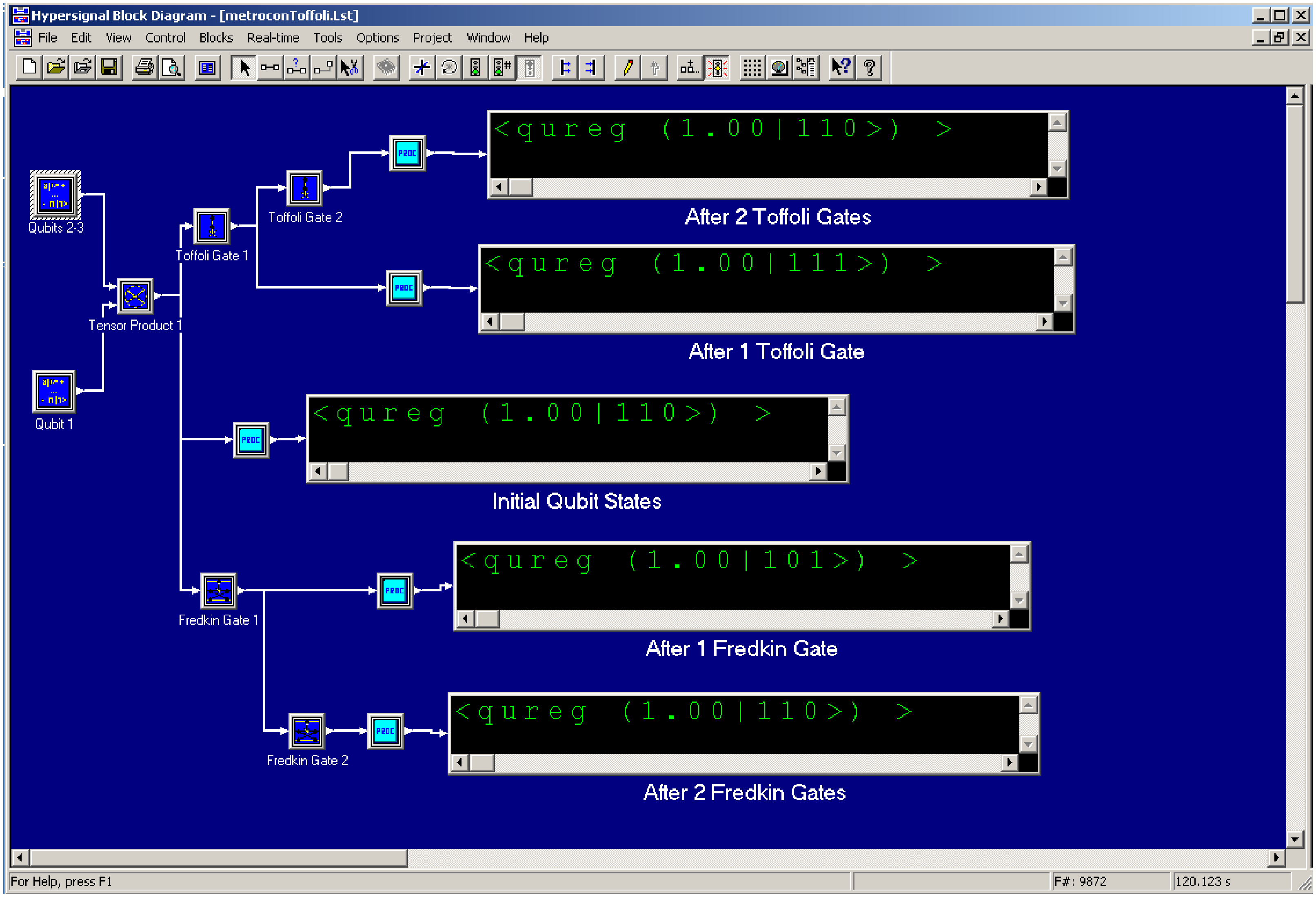
Reversible Quantum Circuits



Gate	Symbolic	Matrix	Circuit
Toffoli = control-control-not	$T * \psi$	$\begin{bmatrix} 1 &&&&&&& \\ & 1 &&&& 0 && \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 1 && \\ & 0 &&&&& 0 & 1 \\ &&&&&& 1 & 0 \end{bmatrix}$	
Fredkin= control-swap	$F * \psi$	$\begin{bmatrix} 1 &&&&&&& \\ & 1 &&&& 0 && \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 0 & 1 & \\ & 0 &&&& 1 & 0 & \\ &&&&&&& 1 \end{bmatrix}$	
Deutsch	$D^* \psi$	$\begin{bmatrix} 1 &&&&&&& \\ & 1 &&&& 0 && \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 1 && \\ & 0 &&&&& i\cos\theta & \sin\theta \\ &&&&&& \sin & i\cos\theta \end{bmatrix}$	

$$\left[|000\rangle \quad |001\rangle \quad |010\rangle \quad |011\rangle \quad |100\rangle \quad |101\rangle \quad |110\rangle \quad |111\rangle \right]$$

Toffoli and Fredkin Demo



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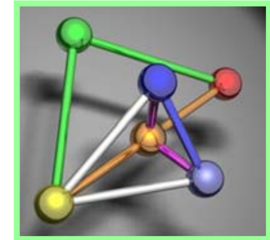
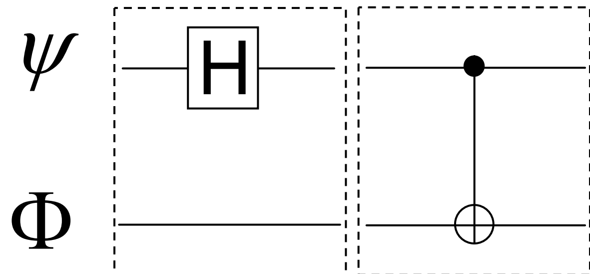
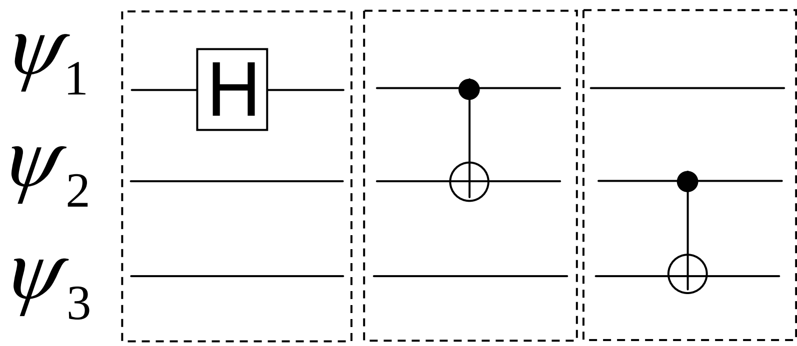


Ebit Generators

Bell Operators

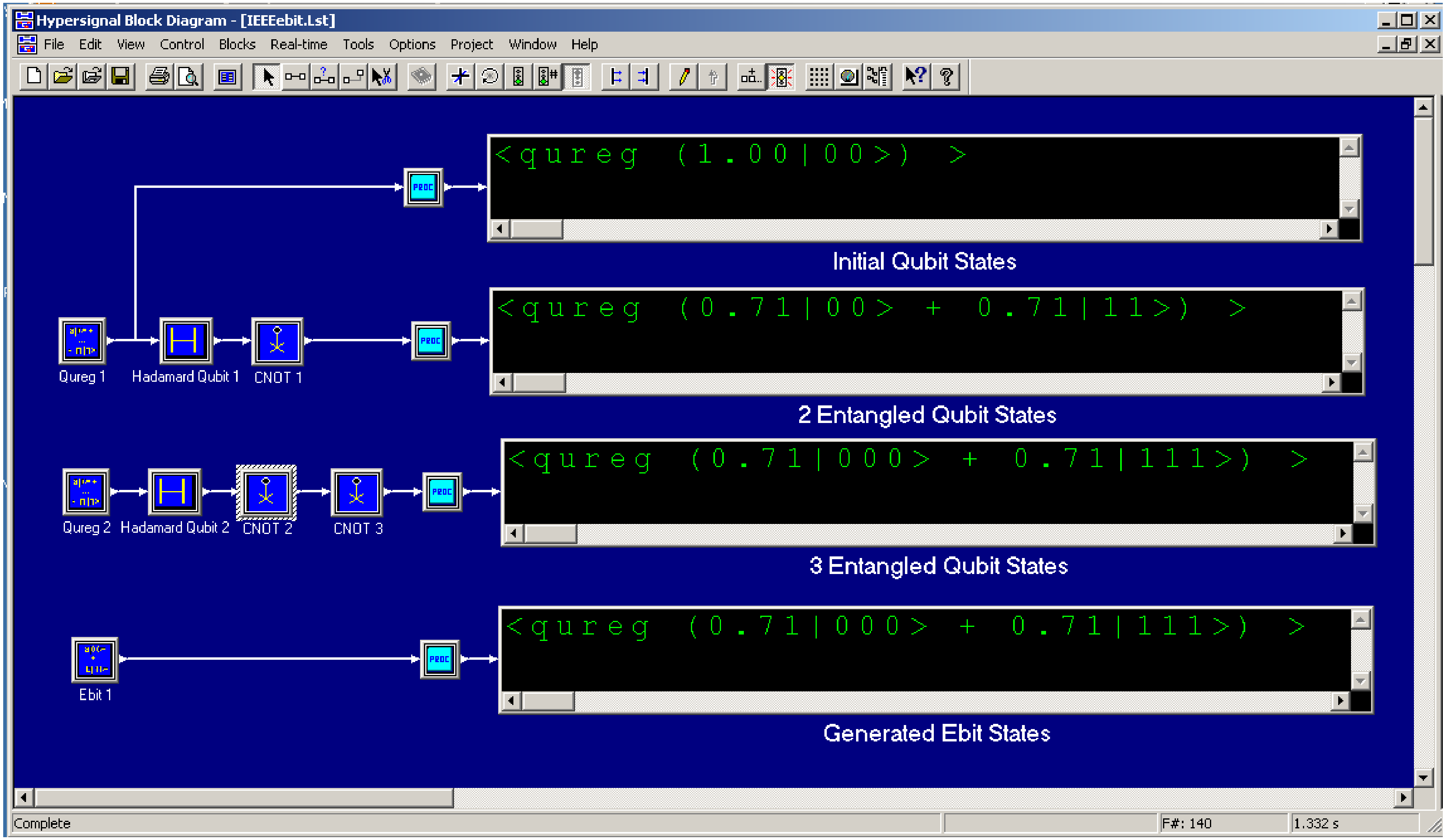


$$\left(H \otimes I_2\right) * Cnot = bell2$$



$$\left(H \otimes I_2 \otimes I_2\right) * (Cnot \otimes I_2) * (I_2 \otimes Cnot) = bell3$$

Ebits Generator Demo



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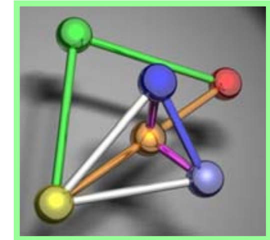
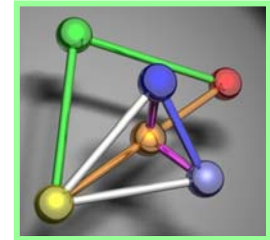
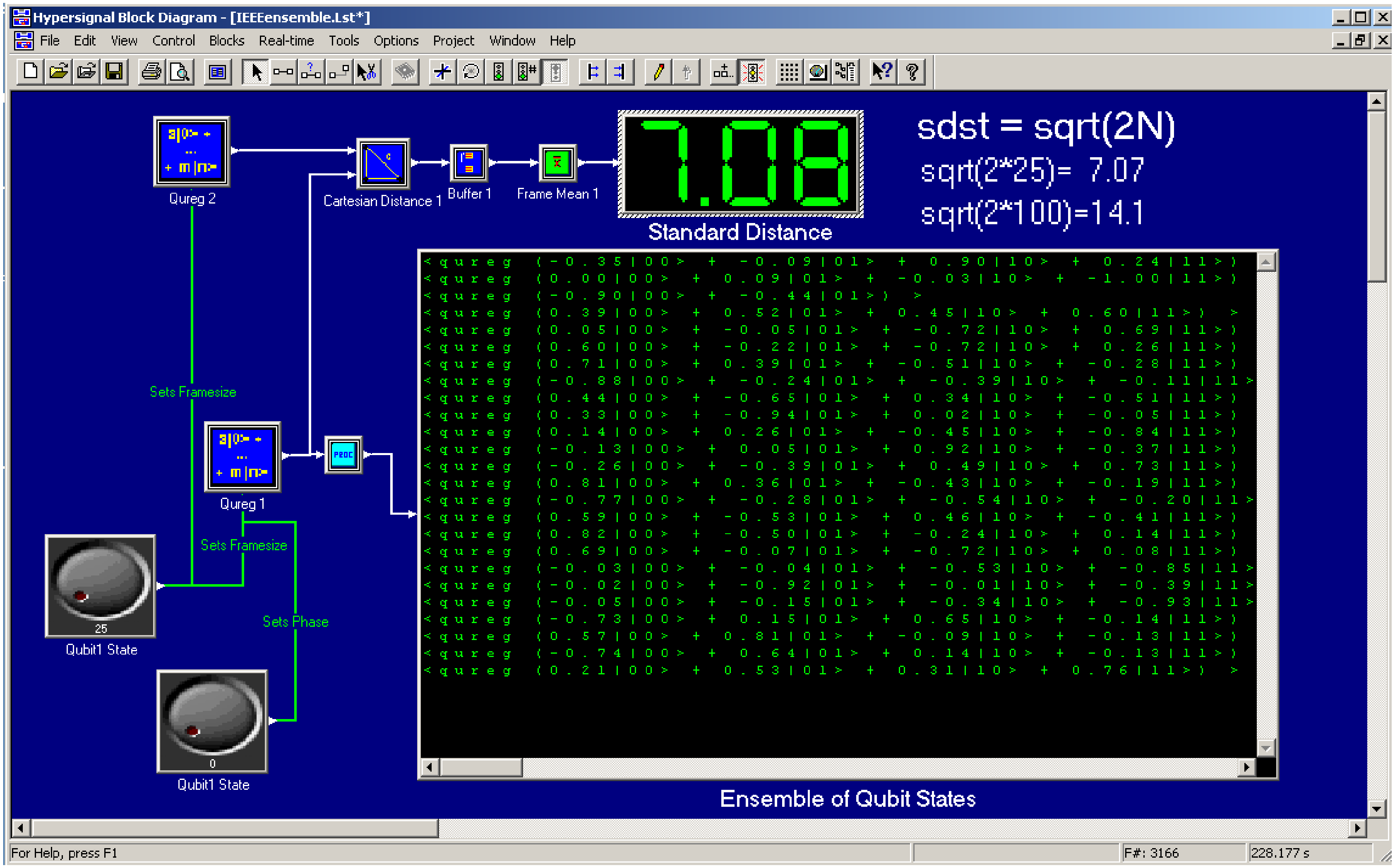
Quantum Ensembles



- N qubits that are arrayed but not entangled
- If random phase for each qubit:
  - Represents a point in high dimensional space
  - Phase Invariant
  - Orthogonal
  - Distance between two random ensembles $\sqrt{2N}$
  - Standard deviation is $\sqrt{1/2}^{q+1}$
  - Same results if each N is a quantum register

Ensembles: Spaces and Points





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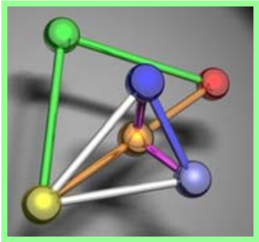
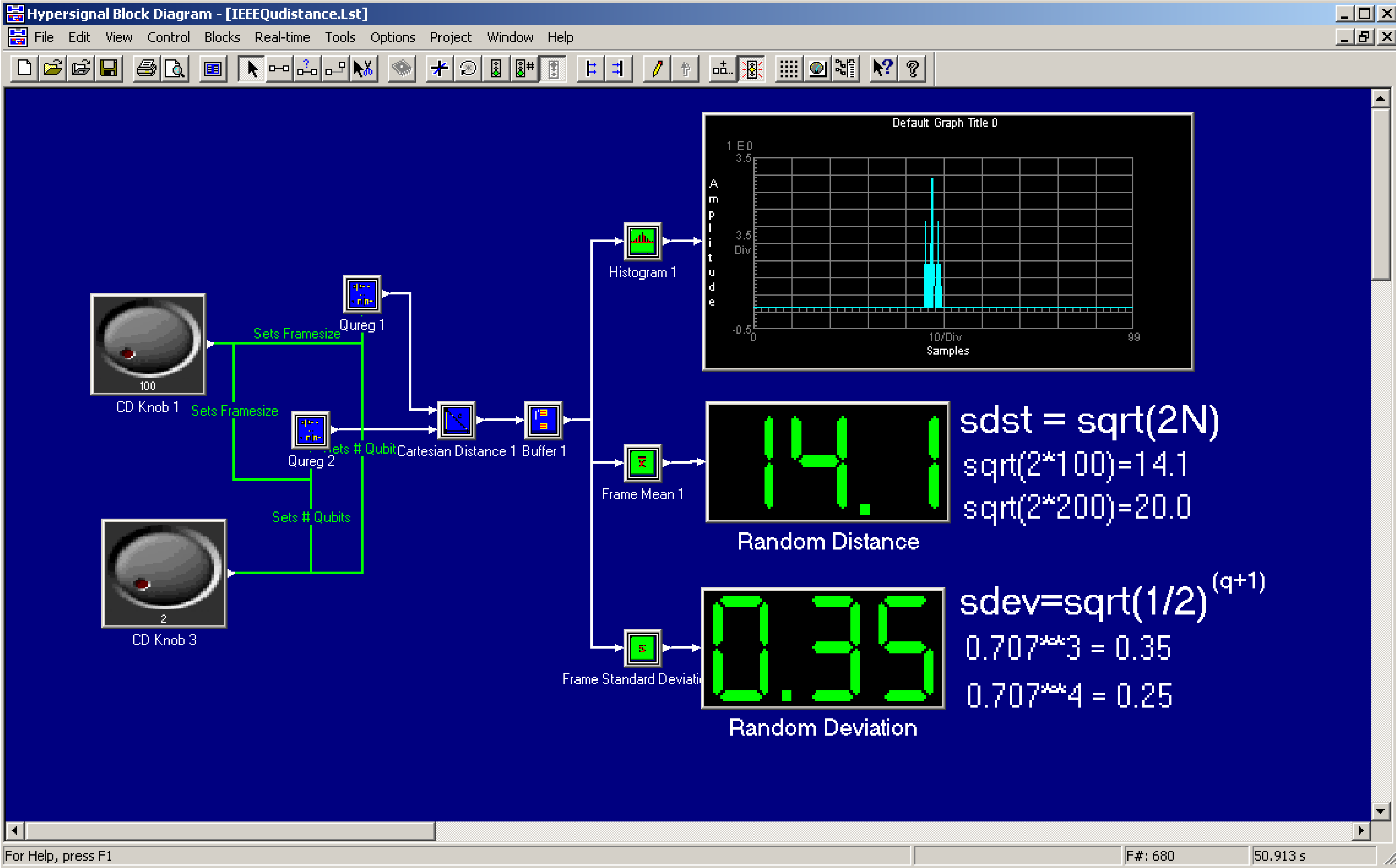
Standard Distance for QuEnsembles





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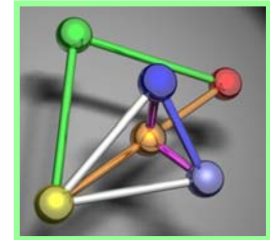
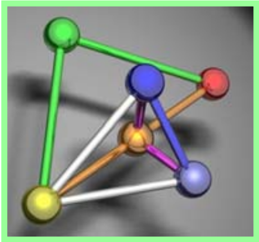
Standard deviation is a independent of N

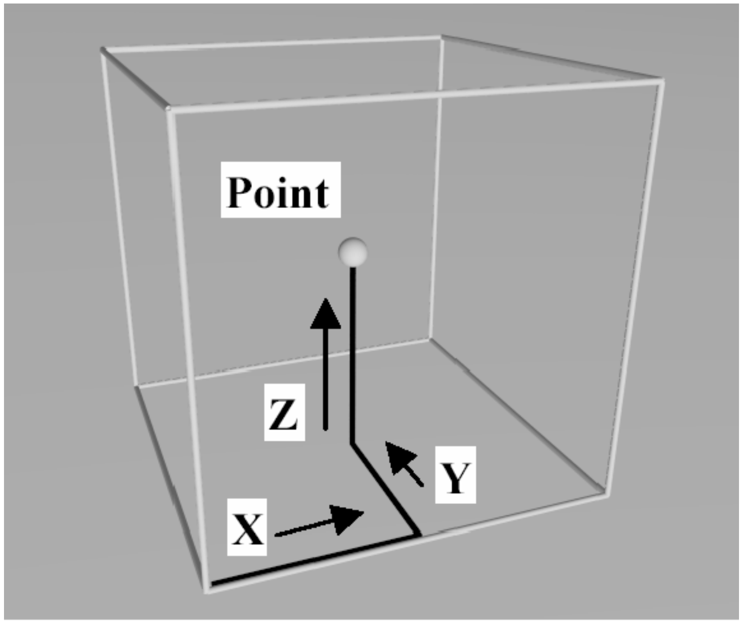
Correlithm Objects



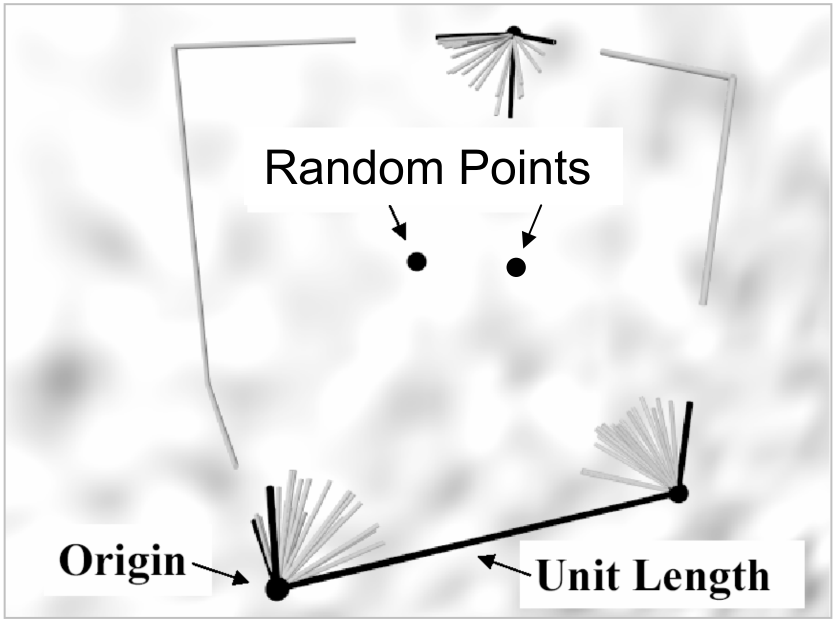
- Points in a Space (Unit cube, Hilbert Space)
- Cartesian Distance between Points
  - Same for all random points/corners of space
    - *Standard* Distance, *Standard* Radius and other metrics
    - Related to field of probabilistic geometry
  - Follows a Gaussian Distribution
    - Mean: grows as $\sqrt{N}$
    - Standard deviation: independent of N
- Key concept/IP of Lawrence Technologies
  - Patents issued and several pending

Correlithm Objects (COs) are Points





1 point in 3 dimensions



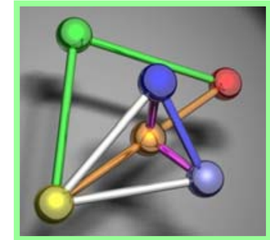
2 points in N dimensions

$$for\ X = [x_1, \ldots, x_N]\ and\ Y = [y_1, \ldots, y_N]$$

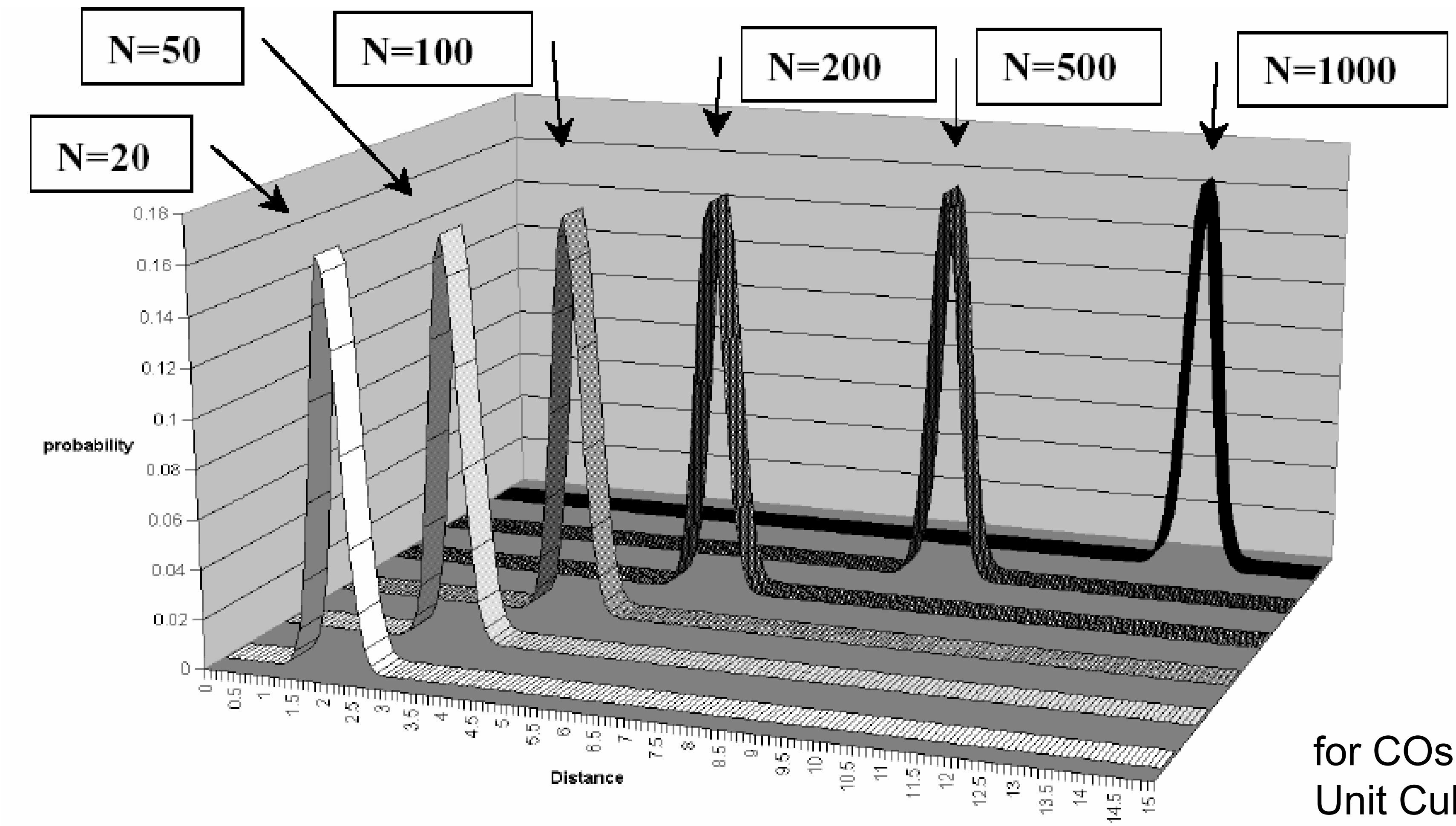
$$dist(X, Y) = \sqrt{(x_1 - y_1)^2 + \ldots + (x_N - y_N)^2}$$

Randomly chosen points are *standard distance* apart.

Cartesian Distance Histograms



"Standard" Distance = $\sqrt{\frac{N}{6}}$ Standard Deviation = $\sqrt{\frac{7}{120}}$



for COs in Unit Cube

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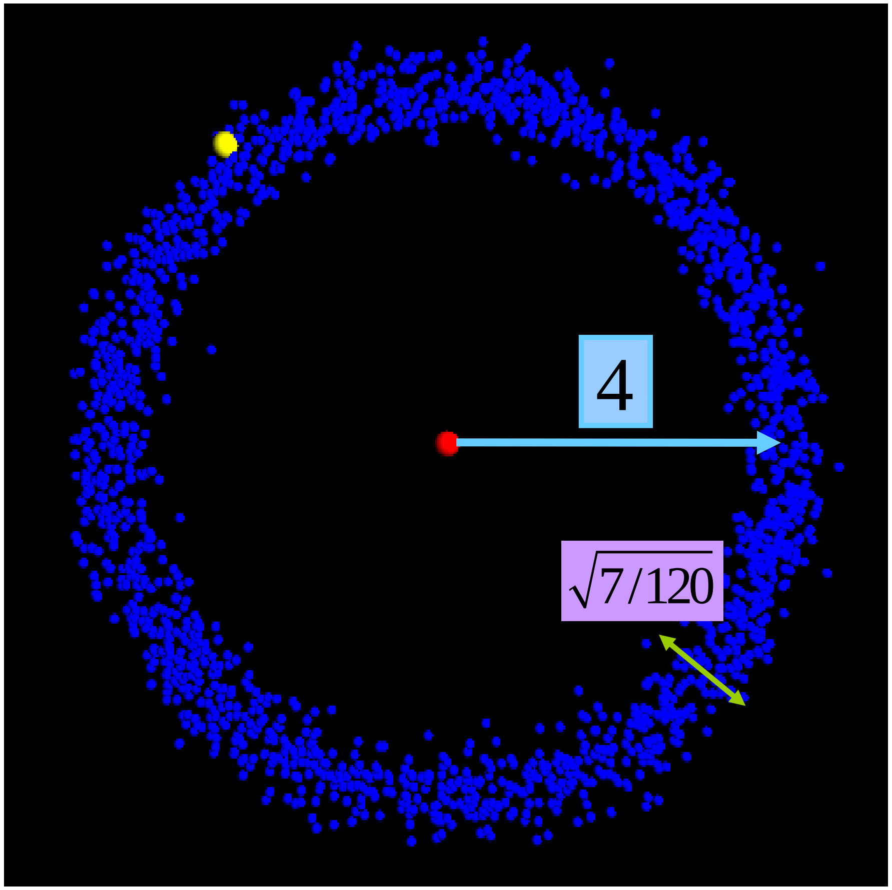
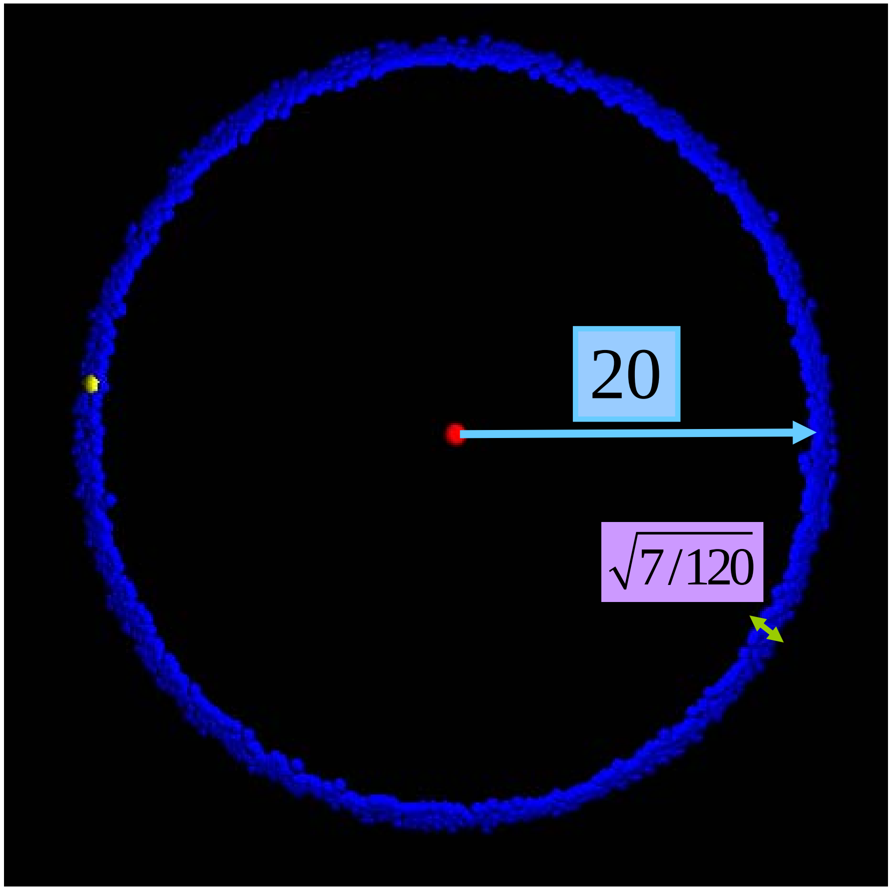
Constant Standard Deviation

For N=96, Standard distance = 4



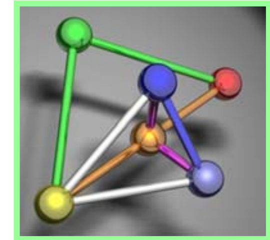
For N=2400, Standard distance = 20



Two plots are scaled/normalized to same relative size

for Unit Cube COs

Standard Distance Metrics



Property	Type	Cartesian Distance	Standard Deviation	Cartesian Distance	Standard Deviation
		Unit Edge		*Unit Radius*	
Edge: length of side	Exact	1	0	$\approx \sqrt{12/N}$	≈ 0
Major diagonal: max corner to corner	Exact	$\sqrt{N}$	0	$\approx \sqrt{12}$	≈ 0
R corner to R corner	Prob	$\approx \sqrt{N/2}$	$\approx \sqrt{1/9}$	$\approx \sqrt{6}$	$\approx \sqrt{4/3N}$
Standard diameter: R corner to R point	Prob	$\approx \sqrt{N/3}$	$\approx \sqrt{1/15}$	$\approx \sqrt{4}$	$\approx \sqrt{4/5N}$
Half diagonal: Half major diagonal	Exact	$\sqrt{N/4}$	0	$\approx \sqrt{3}$	≈ 0
Standard distance: R point to R point	Prob	$\approx \sqrt{N/6}$	$\approx \sqrt{7/120}$	$\approx \sqrt{2}$	$\approx \sqrt{7/10N}$
Standard Radius: Midpoint to R point	Prob	$\approx \sqrt{N/12}$	$\approx \sqrt{1/60}$	1	$\approx \sqrt{1/5N}$

Statistics for random points/corners for Unit Cube COs

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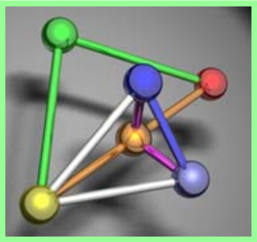
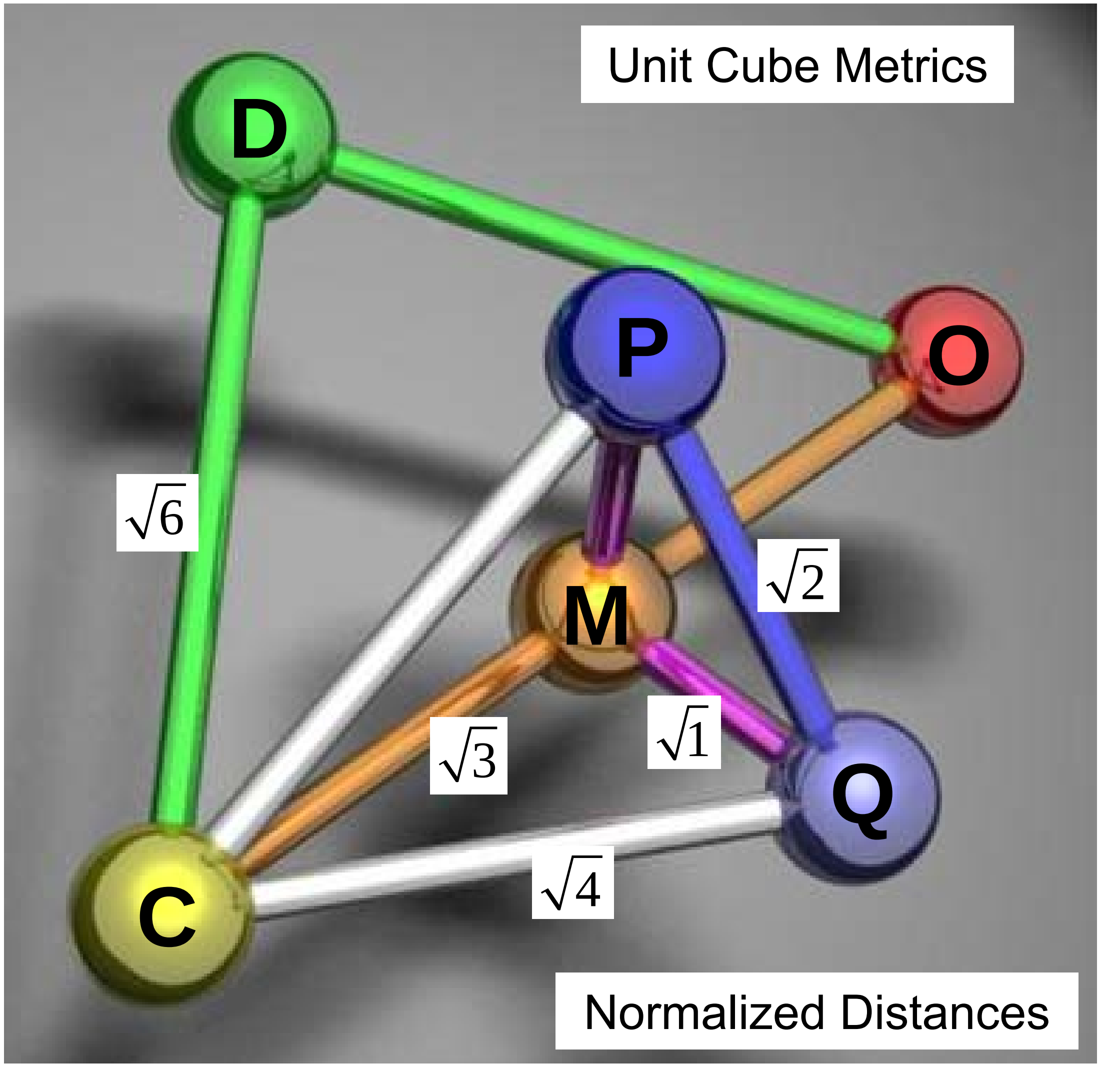
Equihedron Topology

Probabilistically forms high dimensional tetrahedron





Exact Points

C = Corner Reference

M = Mid point of space

O = Opposite Corner

Random Points

P = Random CO 1

Q = Random CO 2

D = Random Corner

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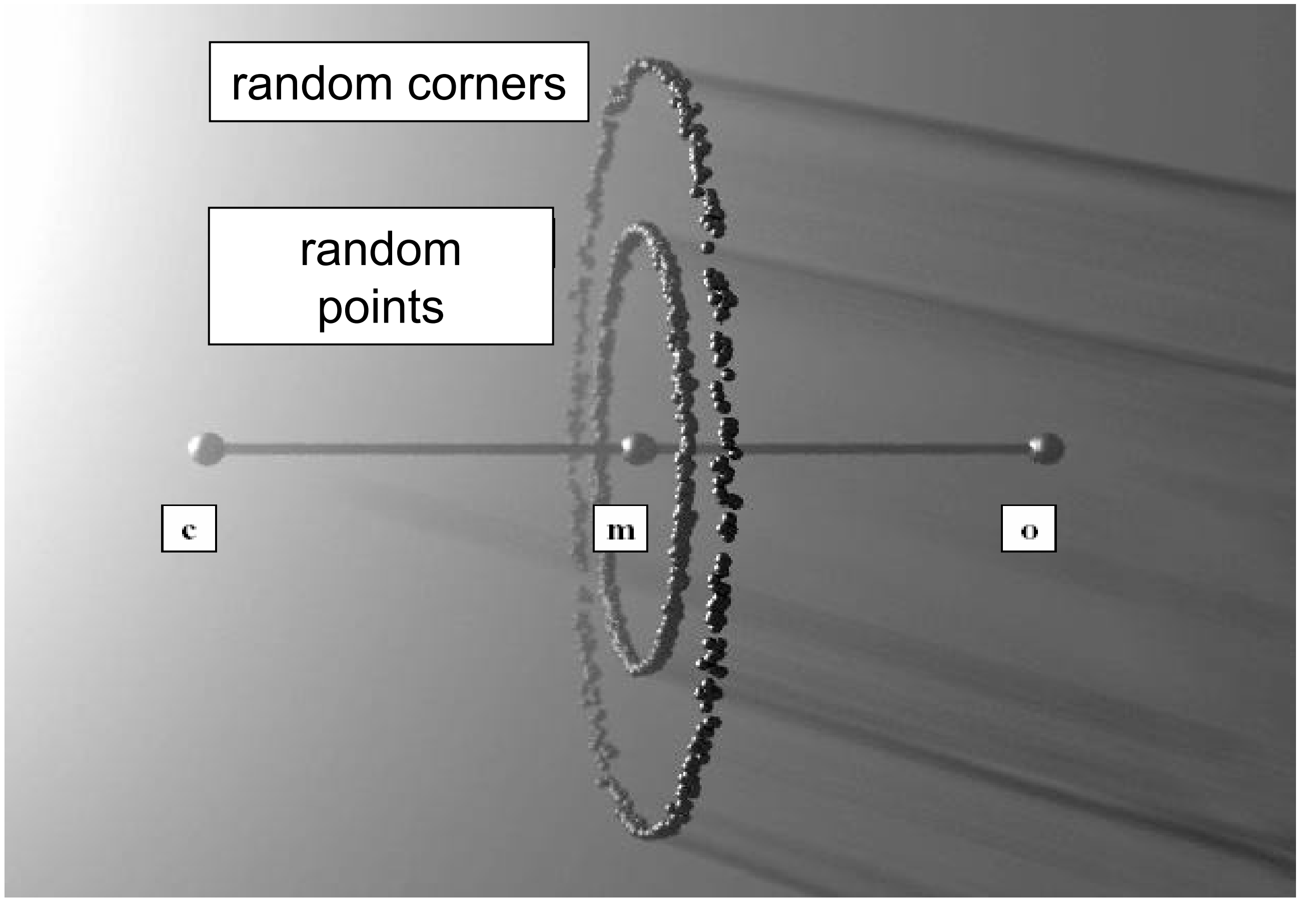
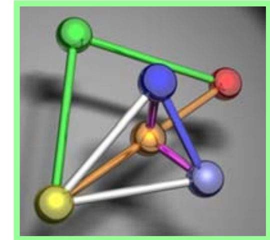
Invariant Metrics



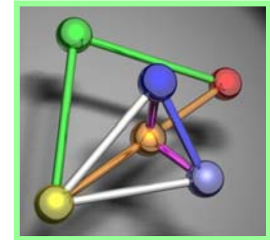
All random CO points are *equidistant* from each other and all random CO points are *equidistant* from center point and all random CO corners are *equidistant* from each other …

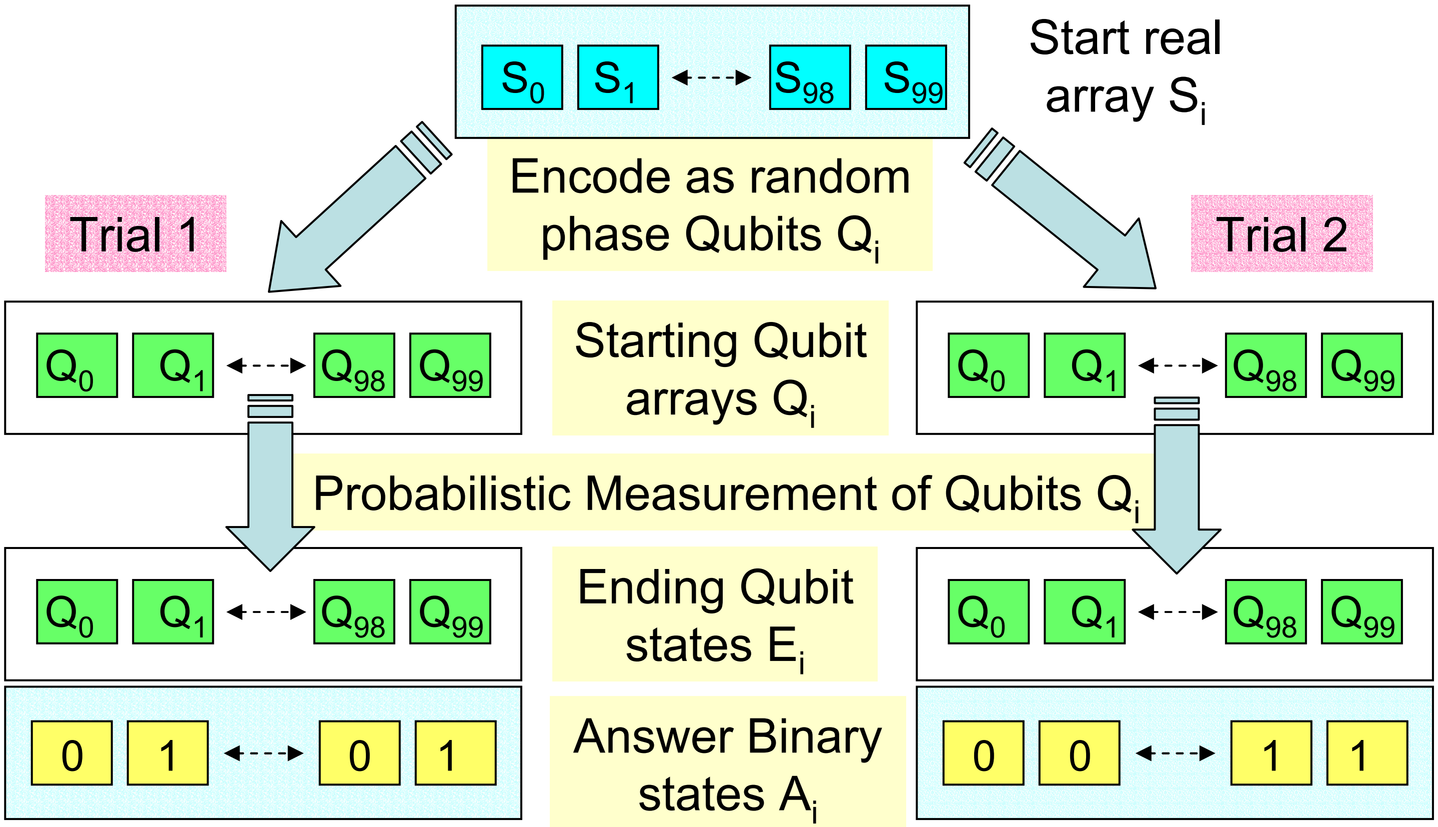
Accessing Quantum COs



- Quantum COs are not directly visible (except thru simulation)
- Measure of QuCOs produces classical CO
  - Answer is binary CO
  - End state is another QuCO
- Multiple trials reveals underlying QuCO
- Measurement is noise injection CO process
- CO tokens survive this process!

COs Survive Measurement



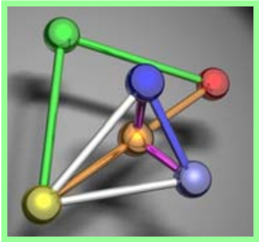


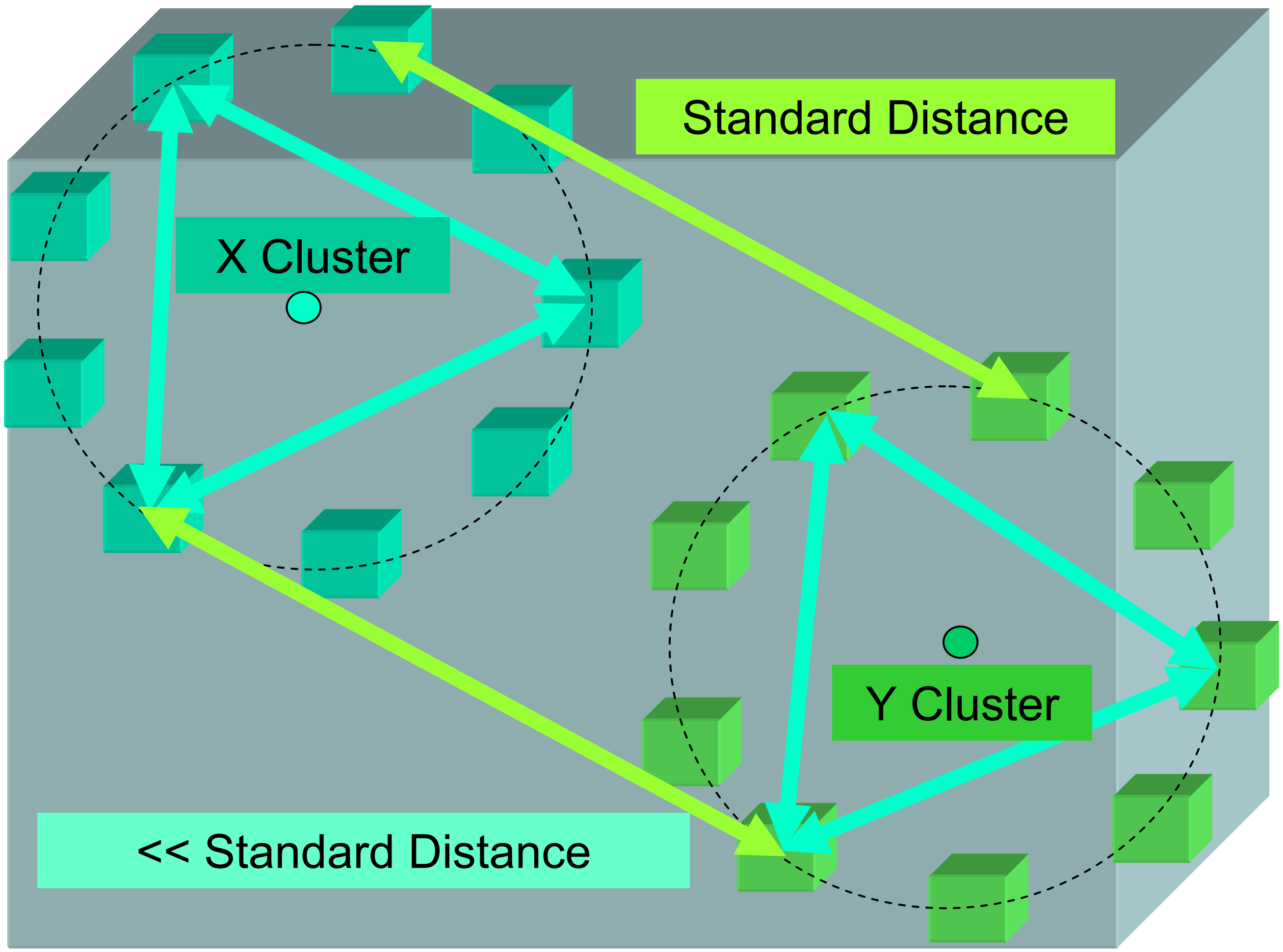
Answers are 50% same from multiple trials of same S_i!!

Repeat Multiple Trials for sets S_i => X and sets S_i => Y

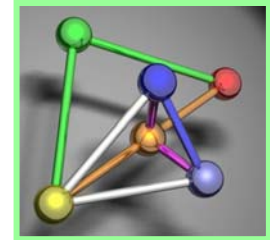
(patent pending)

Topology of COs Survival





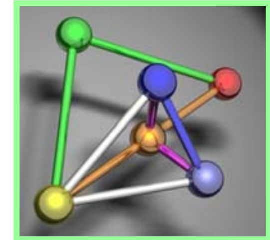
Model Quantum CO process



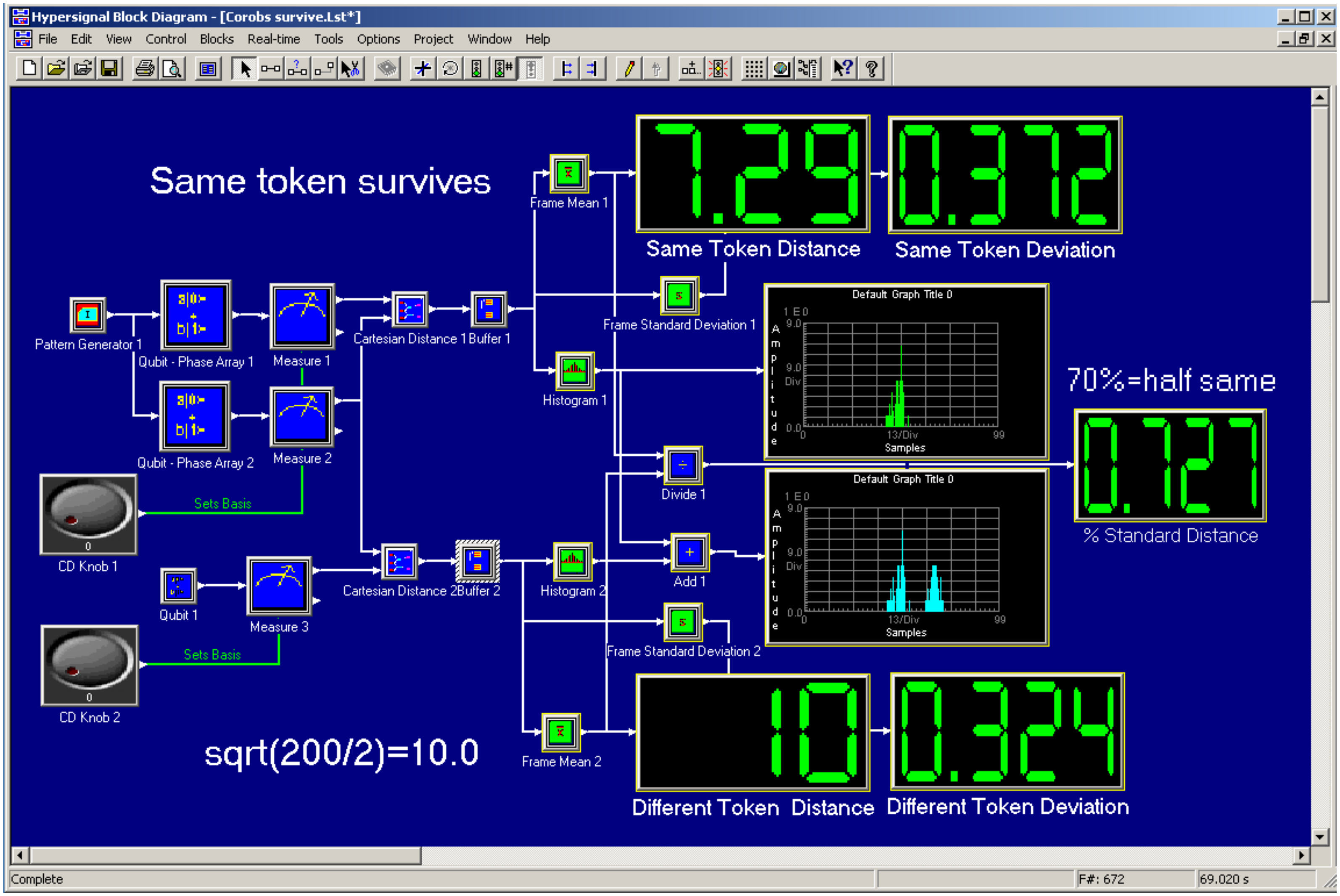
Description of next slide:

- Multiple trials of same CO (top left)
- Multiple trials of random CO (bottom left)
- Make measurements (mid)
- Compare Rand-COs to same CO distances
- Generate histograms (mid)
- Display histograms (right)
- 70% of expected standard distance (right)

Quantum Measurement as COs



Quantum encoded tokens are identifiable after measurement



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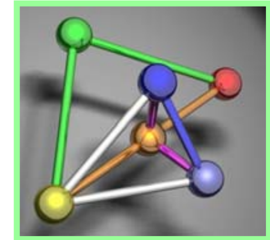
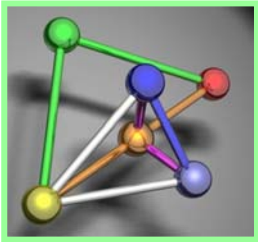
Qu Measurement can be thought of as CO process!

Quantum Ensemble Summary

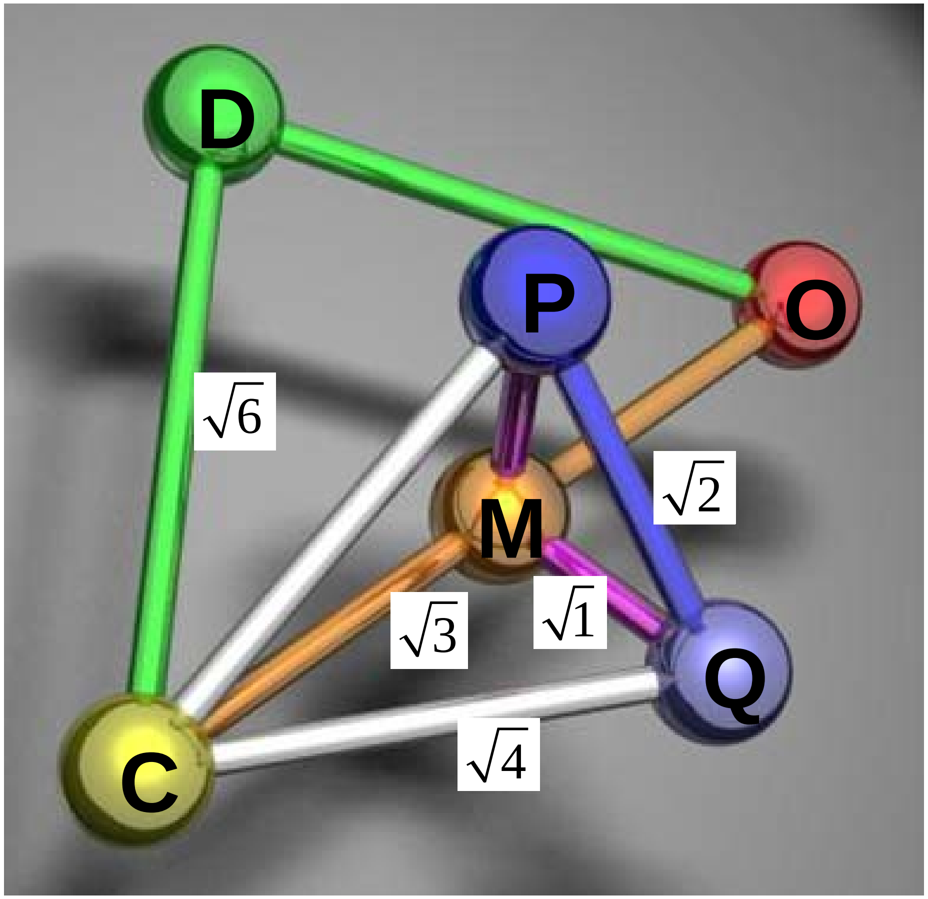


- New tools help explore complex topics
  - Quantum Computation domain
  - Correlithm Object domain
  - Quantum Correlithm Object mixtures
- Quantum & Correlithm theories are related
  - Both depend on probabilities and info. theory
  - Same standard distance for all Qu ensembles
  - Superposition appears in both domains
  - QuCOs survive measurement (patent pending)
    - QuMeasurement cast as correlithm noise process

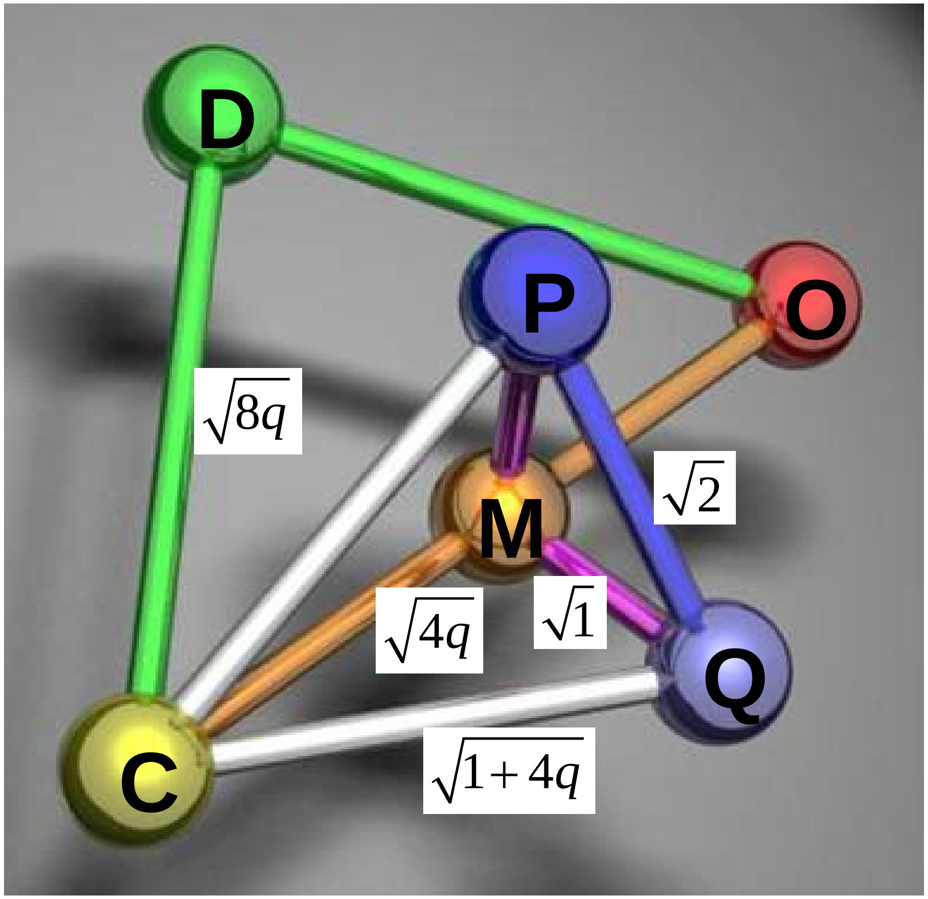
Quantum and Correlithms



Unit N-cube Topology



Quantum Register Topology



Normalized Distances

DJM Sept 29, 2004