Artificial Physics: the soul of a new discipline

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Abstract

Artificial Physics aims to develop a formalism which integrates the concepts of computation and measurement as framed, respectively, by the classical theory of computation and by the quantum theory of measurement. The integrated models of AP are called "autonomous apparatus" or "demons" and they open a new, more general route to the understanding and implementation of computation at the quantum level. The (in)formal basis for the development of a theory of such devices is presented together with a few exemplary instances of problems, found in the gap between physics and computation, to which its application may offer new insight.

1 Introduction

> *"The computing process, where the setting of various elements depends upon the setting of other elements at previous times, is closely akin to a measurement. It is difficult, however, to argue out this connection in a more exact fashion. Furthermore, the arguments concerning the measurement process are based on the analysis of specific models (as will some of our arguments about computing), and the specific models involved in the measurement analysis are far from the kind of mechanisms involved in data processing. In fact the arguments dealing with the measurement process do not define measurement very well, and avoid the very essential question: When is a system* A *coupled to a system* B *performing a measurement? "*

1.1 The dawn of the reversible computer

The above quotation comes from the introduction of a remarkable paper [20] with which Rolf Landauer inaugurated, thirty years ago the modern inquiry into the nature of computation as a physical process. Today we are all familiar with the outcomes of this quest: Landauer's principle which establishes the lower limit of energy dissipation for computers with irreversible components and the more striking discovery of the conceptual possibility of *reversible computation* by Bennett[6], and by Fredkin and Toffoli[17]. The latter suggests a deeper relation between physics and computation than it has been historically aknowledged. Traditionaly regarded as a branch of mathematics computation is understood by physicists as a tool for analysis, modeling or simulation[12] of complex physical systems. However, reversible universal computation offers the option of picturing these familiar employments as "irreversible macro-computations" ultimately composed by reversible steps much like Physics understands the nature of the natural processes being modelled or analyzed. Thus most of the conceptual examples of reversible computers which have been proposed (billiard-ball computers, Brownian computers, etc...) are simple physical systems but a-priori more realistic than Turing machines. While the actual depth of the analogy is still being accessed[21] it seems clear that, in the reversible context, computation represents a qualitative interpretation domain for the universal *evolution* laws assigned by physics to the description of dynamical systems. Reversibility also makes it tempting to regard physical systems as enacting "natural" or extensional computation where the dynamical laws appear as emergent phenomena[32].

1.2 Quantum Computers: the zeroth generation?

In spite of considerable controversy, reversible computation is today a well established concept, as one would gather by the numerous devices (both conceptual and pragmatic) which have been proposed to implement it[22]. Several authors during the eighties have taken one step further and sought to extend the work of Landauer, Bennett, Fredkin and Toffoli conceiving and exemplifying computational processes carried out by suitably designed quantum systems. These efforts were fueled by the hope that suitably designed quantum systems could be made to implement reversible computation while avoiding some of the problems that make the classical versions impractical, namely the destructive effects of fluctuations and material defects. Yet this "zeroth generation of quantum computers", initiated by Feynman[16] and Benioff[5] and lately spearheaded by Deutsch[15], promises to open the way for an understanding of natural computation at a fundamental physical level.

It remains however highly debatable whether the past 30 years have bought a comparable clarification to the matter of Landauer's original query, namely *the relation between computation and measurement.* As it

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is well known the understanding of the measurement process as a fundamental, and thus reversible, physical process is hampered by the so-called *quantum measurement problem* which seems to suggest that there is a subtle irreversibility at the fundamental level, responsible for the actualization or objectification of classical observables[26]. The 0th generation proposals largely ignore this and other paradoxical aspects of Quantum Theory[27]. Thus twenty six years after his original insight Landauer still gave a tentative assessment of the relations between computation and measurement:

> *"It is sometimes assumed that, even if information transfer within the computer can be done without minimal dissipation, this does not apply to the input and output; a measurement at this point is essential. [...] Do we need a dissipative measurement to signal the completion of a program ? That, most certainly, is one possibility."*

1.3 Demonology and Eschatology: The Artificial Physics alternative

In this paper we sketch a new approach to the problem of conceptualizing and implementing computation at a fundamental level deploying a new program of research which takes up Landauer's challenge directly, seeking to develop a unified view of computation and measurement. I call this program **Artificial Physics (AP)** both to enforce the contrast with "natural" computation and to stress some of its high-end aims which replay some of the themes once embraced by Artificial Intelligence. The central concept of **AP** is that of an *autonomous apparatus* or *demon*, a computational device also capable of performing measurements.

The paradigm of a demon is, of course the famous construction of Maxwell which has been haunting four generations of physicists[24]. As is well known the latter day exorcism of this creature owes much to its reassessment as a computing device[8] and to Landauer's understanding of the notion of "entropy of erasure" as a form of information disposal. In the same way as Maxwell's demon illustrates an (apparent ?) violation of the second law of Thermodynamics leading to a perpetual motion (i.e. non-halting) engine the more generic demons of **AP** will embody its aim to build a theoretical understanding of physical laws by constructing instances of their violation and mapping them to infinitary discrete computation-like processes. The obvious qualitative paradigms come from the fundamental no-go theorems of the theory of computation but the analogy also extends to some quantitative results from space and time complexity theory (demons of the 1st kind). In the later context its worth remembering that demons have been used for some time in Computational Physics[13, 14] in much the same spirit as they are invoked here, that is, as devices for implementing local violations of energy conservation in the context of microcanonical update algorithms. Incidentally, Creutz demons which drastically accelerate equilibration algorithms, attest to the position assumed by **AP** that not all demons are evil[1], in spite of their long (cartesian) tradition of conspiracy against the laws of Physics [33].

In this paper I introduce the **AP** point of view through a a motivating example drawn from the history of reversible computation[7]. I will show how such example translates to the quantum domain introducing an old but little known alternative to the von Neumann description of Quantum Measurement. In the next section I will sketch the elements of the constructive definition of "demon" adopted by **AP** and the demon hierarchy accommodated by this definition. The last section describes what I believe to be the simplest quantum demon that may aspire to universality in the enlarged sense of **AP.** I will conclude with some remarks pointing to the *eschatology* of Artificial Physics, more precisely the business of giving meaning to this notion of Universality and the more mundane concerns of how one can simulate and ultimately build elementary demons[23].

2 Computation and/or measurement.

2.1 Unitarity and Reversibility

The little common ground to be found between the "zeroth generation" approaches to Quantum Computation consists of the association, incisively spelled out by Peres (1985), of the state vector or wavefunction with the software of the quantum computer and its observables (or self-adjoint operators) with the hardware. This assignment is dictated by the 'natural' compulsion to identify the computation with the linear, unitary (and thus time-reversible) evolution $\mathbf{U}$ of the state vector, encapsulated in the Schrödinger equation. Thus a time-discrete unitary evolution (written in terms of finite differences between 'future' $|\mathtt{t}+1\rangle$ and 'present'$|\mathtt{t}\rangle$ states):

$$|u(\mathbf{k}), \mathtt{t}+1\rangle := \mathbf{U}_1|\mathbf{k}, \mathtt{t}\rangle \simeq (1 - iH_1)|\mathbf{k}, \mathtt{t}\rangle \quad (1)$$

immediately suggests itself as a choice candidate for the schematic of a "quantum computational step" i.e. a discrete first order "numeric" dynamics driven by a time independent Hamiltonian H_1 to be fashioned from reversible primitives. A full sequential computation of a function $F(\mathbf{k})$ of N arguments in $\mathbf{T}$ steps, is than understood as a global map:

$$|\mathbf{k_O}, 0, 0\rangle \longrightarrow |\mathbf{k_O}, F(\mathbf{k_O}), \mathtt{T}\rangle$$

where the initial state $|\mathbf{k_O}, 0\rangle$ vector summarizes $N+1$ possible input arguments. This process must be supported by a physical evolution of this system enacted by successive applications of an Hamiltonian which includes provision for manipulative instructions (the program). However the zeroth generation's reliance

[1] Demons of Maxwell kind are associated with perpetual motion engines of the 2nd kind which violate the second law of thermodynamics. Creutz's demons are of the 1st kind as they represent violations of the 1st law.

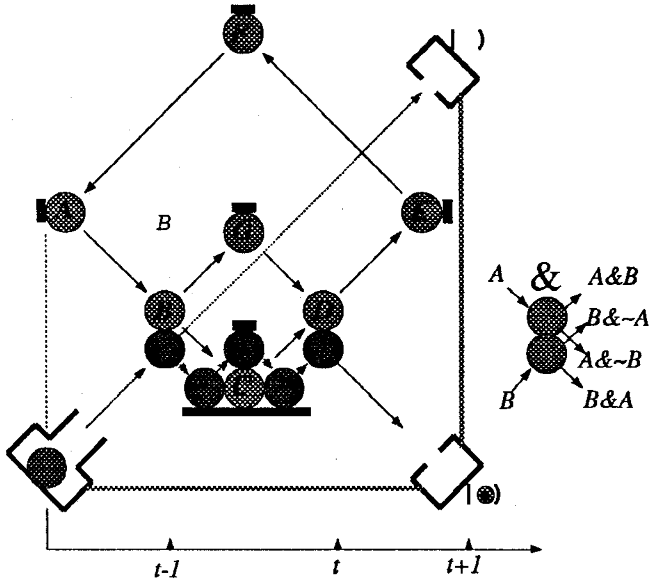

Figure 1: Reversible measurement on a Billiard Ball Model of Computation using **AND** gates.

on the unitary evolution scheme (1) inverts the logical order of arguments that led to reversible computation. Classical computational schemata inspired by (Newtonian or Hamiltonian) dynamics which are linear in their time dependence and can be represented by a step assignment of the type

$$|\mathbf{k}, \mathbf{t}+\mathbf{1}) := \mathbf{V_1}(\mathbf{i}, \mathbf{j}, \mathbf{k}, ...)|\mathbf{k}, \mathbf{t}) \qquad (2)$$

where $\mathbf{V_1}$ represents a nonlinear step potential operator (rule) with discrete space dependence, are typically *not* reversible. Fredkin has proved though that, for a large class of such rules one can obtain a reversible rule simply by *subtracting the past*, that is:

$$|\mathbf{k}, \mathbf{t}+\mathbf{1}) := \mathbf{V_1}|\mathbf{k}, \mathbf{t}) \ominus_n |\mathbf{k}, \mathbf{t}-\mathbf{1}) \qquad (3)$$

where the $\ominus_n$ stands for cyclic subtraction modulo n (number of states). This transformation raises the rule to second order, mapping it to the second finite difference, (which interprets the differential operator d^2/dt^2), while $t \longrightarrow -t$ can be seen to lead to an equivalent time-reversed rule.

2.2 Classical reversible measurement: an example

Fredkin's transformation is what makes the possible the *classical* picture of a reversible *non-demolition* measurement conceived in fig. 1 for a billiard-ball computer model. The objective is to determine the presence or absence of the light shaded ball within a delimited region, i. e., to measure the the "presence" operator Δ with eigenvalues 1 and 0. The detection is undertaken via the injection of another darker ball which will accuse the presence of the previous one rebounding from a collision at time $t-1$ which changes its path slightly, from ABC to ABG but restores it to the original through a second collision at time t. The final step in the measurement process takes place at $t+1$ when either of the counters, labeled by the eigenstates of the measured variable Δ, $|\odot)$ or $|\ )$, clicks indicating whether a particle is present or not. The complete measurement can be described by the difference equation:

$$M^B_\Delta|A;\mathbf{t}+\mathbf{1}) := N^{B'}_A|A';\mathbf{t}) \ominus_2 N^{B''}_A|A'';\mathbf{t}-\mathbf{1}) \quad (4)$$

where the variables A, B labeling the states of the two elastically colliding balls are respectively the signs of their transversal momenta $A = \mathrm{sign}\mathrm{P}^{\mathrm{dark}}_{\mathrm{y}}$ and $B = \mathrm{sign}\mathrm{P}^{\mathrm{light}}_{\mathrm{y}}$. The operator M^A_Δ describes the *measurement* of the quantity Δ on a ball prepared in state A while the "*pre–measurement*" operator:

$$N^B_A = \sum_{C=\pm} V_{1/2} P^{\&}_C V_{1/2}$$

is the product of the projection operator implemented by the reversible AND gate operator $P^{\&}_C = |C)(C|$ (with $C = A+B$ or $C = A-B$) together with the $V_{1/2}$ **half–step** operator which "evolves" the ball's motion between collisions. The result of the measurement is fixed by the values of the final y component of the dark ball after the two collisions at $\mathbf{t}$ and $\mathbf{t}-\mathbf{1}$. The two final state options $|\ )$ $|\odot)$ are pointed to respectively by the $|A > 0)$ and $|A < 0)$ states of the transversal momentums sign of the dark ball while the original state of the light ball $|B = B'')$ is fully restored. The above expression is just a 'Fredkin–like' rewriting of:

$$M^{B'}_\Delta|A';\mathbf{t}) := N^B_A|A;\mathbf{t}+\mathbf{1}) \oplus_2 N^{\dagger B''}_A|A'';\mathbf{t}-\mathbf{1}) \quad (5)$$

with $N_A N^\dagger_A = 1$ and $\oplus_2$ the binary XOR operation. But then it is clear from both assignments that the 'future' correlation of final states is obtained by an *extrapolation* on both the present and past events.

This example illustrates the relation between Fredkin's transformation and the construction of a reversible measurement prescription. Clearly the reversibility of the whole measurement process depends upon the reversibility of the past and present interaction events but the crucial outcome which is taken to insure the reversibility of the whole process is the restoration of the original state of motion by the joint effect of the two events. The measuring situation is totally symmetric with respect to the two interacting systems. Nothing would in principle prevent us from enclosing the dark ball in its own two–dimensional box providing it with its own set of reflective walls. What distinguishes the measuring device is the final choice of which ball to look at.

2.3 Descriptions of Measurement in Quantum Mechanics

These considerations are relevant for any attempt to extend this measurement model to Quantum Mechanics. Several schemes for Quantum Non-demolition Measurements (QMD) have been proposed[9, 11] over the last decade which implement measurements that preserve orthogonal states. These are low noise measurements which are typically *not* reversible and thus only remotely comparable to the situation envisaged above. As mentioned before most textbook descriptions of quantum mechanical measurement theory emphasize an intrinsic dissipative nature to quantum measurement which is part of the orthodox lore of Quantum Theory.

The standard model of quantum mechanical measurement is due to von Neumann and it describes measurement in time asymmetric stages (Fig.2(a)). Using an extension of the language of the previous example where the classical states $|K\rangle$ are replaced by quantum states $|\Phi\rangle$ we can describe the von Neumann scheme by:

$$M_c^{\psi}|\Phi_C;\mathbf{t}+1\rangle := N_b^{\psi}|\Phi_B;\mathbf{t}\rangle \oplus_2 N_a'^{\phi}|\Phi_A;\mathbf{t}-1\rangle \quad (6)$$

where the two partial measurements N and N' can be further analyzed as: (1) a preparation $N_a'^{\psi} = P^{\psi_i,\Phi} = |\psi\rangle|\Phi_A\rangle\langle\Phi_A|\langle\psi|$ of the initial states of both system and apparatus and (2) a delimited interaction N_B^{ψ} between the two subsystems which can be decomposed into three stages:

$$N_B^{\psi} = \sum_{\phi'} P_B^{\psi} U_1 P_A^{\phi'}$$

with $P^{\phi'} = |\phi'_a\rangle\langle\phi'_a|$ being a complete set of projection operators pertaining to a $\{|\phi'_a\rangle\}$ basis of the target system and U_1 a unitary evolution step operator associated with the interaction (as in (1)). In this situation the von Neumann model prescribes a specific relation (projection postulate) between each possible final state (understood as an eigenstate $\phi_a\rangle$ of A and the initially prepared ψ_i namely:

$$|\psi_i\rangle \mapsto |\psi_f\rangle = |\phi_a\rangle\langle\phi_a|\psi_i\rangle / \sum_{\phi'_a}\langle\phi'_a|P_B^{\psi}|\phi'_a\rangle$$

It should be pointed out that in spite of its prevalence the von Neumann model is by no means a general description of quantum measurement[3]. It amounts to a definition of a particular class of ideal measuring instruments while others have been proposed with higher generality and realism[10]. Its time asymmetric nature in particular is not required by Quantum Mechanics. There is indeed an alternative time symmetric description of quantum measurement introduced by Aharonov, Bergman and Leibowitz[1] (Fig.2(b)) which bears more than a passing resemblance to the

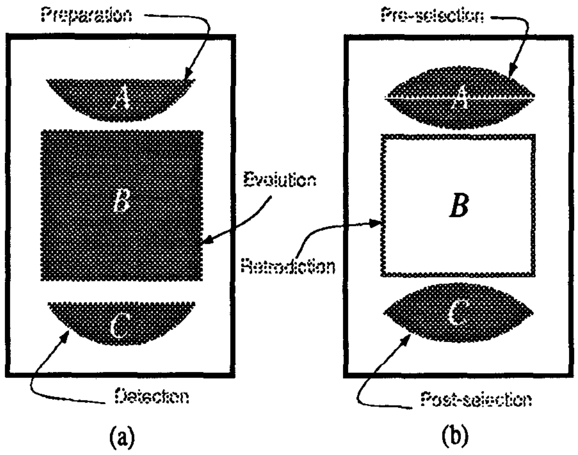

Figure 2: Measurement descriptions: (a) von Neumann and (b) Aharonov–Bergmann–Leibowitz.

Fredkin example. In this picture both the final (future) and initial (past) states are specified by independent measurements and the intermediate (present) probability distribution is univocally *computed by an interpolation* formula imposed by the properties of conditional probabilities (Bayes Theorem). In the general context of the intended measurement of the physical quantity B (or any function of it) of system S the ABL description can be sketched as:

$$M_b^{\psi}|\Phi_B;\mathbf{t}\rangle := \Lambda_c^{\psi}|\Phi_C;\mathbf{t}+1\rangle - \Lambda_a^{\dagger\psi}|\Phi_A;\mathbf{t}-1\rangle \quad (7)$$

with partial measurements:

$$\Lambda_c = P_c^{\psi} U_1 P_b^{\psi'} \quad \text{and} \quad \Lambda_a^{\dagger} = P_a^{\psi''} U_1^{\dagger} P_b^{\psi'}$$

In this case the ABL interpolation formula computes the probability of obtaining a value B' at time $\mathbf{t}$ as:

$$\Pr(B = B') = |\langle \mathbf{t}; \phi_{b'}|\phi_{b'}; \mathbf{t}\rangle|^2 = \frac{|\langle t-1, \Phi_A|\Lambda_a^{\dagger} P_b^{\psi}\Lambda_c|\Phi_C, \mathbf{t}+1\rangle|^2}{\sum_{b'}|\langle \mathbf{t}-1, \Phi_A|\Lambda_a^{\dagger} P_{b'}^{\psi}\Lambda_c|\Phi_C, \mathbf{t}+1\rangle|^2} \quad (8)$$

with $P_{b'} = |b';t\rangle\langle t;b'|$. This expression is considerably more general than (5) as the quantities A, B, C need not be instances of the same physical observable as they did in the two dimensional context. They may be different components of the momentum, for example, or even different observables altogether. But (5) is also generalizable to higher dimensional boxes and respective momentum components.

2.4 Generalizing the classical example

One can readily appreciate the analogy by generalizing (5) from a deterministic to a *classic probabilistic* computational context. For this we consider an ensemble of similarly prepared detection experiments and ask for the probability of obtaining each of the final values $|F) = \{|\ ), |\odot)\}$ or $(F|M^B_\Delta|A;\mathtt{t}+1)$ of final states. Our use of Dirac notation simplifies this task: we need only to introduce the operator $P^t_C = |C;\mathtt{t}) \oplus (\mathtt{t};C| = P^{\&}_{C'} + P^{\&}_{C''}$ and the "conjugate" measurement to (5):

$$(\mathtt{t}+1;A|M^B_\Delta := (\mathtt{t};A'|N^{B'}_A \oplus_2 (\mathtt{t}-1;A''|N^{\dagger B''}_A \quad (8*)$$

to obtain the normalizing factor corresponding to the denominator of (7). The resulting probability expression is simply:

$$\Pr[\Delta(B)] = (\mathtt{t}+1;A|A;\mathtt{t}+1) := \frac{(\mathtt{t}-1,A''|V_{1/2}P^t_C V_{1/2}|A',\mathtt{t})}{\sum'_C(\mathtt{t}-1,A''|V_{1/2}P^t_C V_{1/2}|A',\mathtt{t})} \quad (9)$$

The B dependence in (11) hidden in the definition of P^t_C can however be explicitated thanks to the **factorizability** of classical states, that is: $|C) = |A+B) = |A)|B)$. Thanks to it and to the orthogonality of value and time dependence $(A|B) = 0 = (t|t')$ we can write (11) as:

$$\Pr[\Delta(B)] = \frac{(\mathtt{t}-1,A''|V_{1/2}P^t_{B'}V_{1/2}|C',\mathtt{t})}{\sum'_C(\mathtt{t}-1,A''|V_{1/2}P^t_{B'}V_{1/2}|C',\mathtt{t})}$$

where $P^t_{B'} = |B')(B'|$ is the appropriate restriction of P^t_C to the B state subspace. By the same token we should point out that the **non–factorizability** of quantum states implies that (7) has one further metamorphosis with no classical correspondence. Both past and future quantum states can very well be *superpositions* of system and apparatus states so that one can write in full generality:

$$\Pr(B=B') = |\langle\mathtt{t};B'|B';\mathtt{t}\rangle|^2 = \frac{|\langle\mathtt{t}-1,\Psi''|P_{A''}U_1P^t_{B'}U_1P_C|\Psi,\mathtt{t}+1\rangle|^2}{\sum_{B'}|\langle\mathtt{t}-1,\Psi''|P_{A''}U_1P^t_{B'}U_1P_C|\Psi,\mathtt{t}+1\rangle|^2} \quad (10)$$

Granted that time–symmetry of probability distributions is not the same as reversibility, we can at least ask whether the computational outcomes may in some case be equivalent. In the following section we collect the elements necessary for putting together a physical model of what is simultaneously a measurement device and a computer and later use them to synthesize one such device which implements measurements in the ABL fashion.

3 Crafting Demons

3.1 Models of computation.

The most straightforward way to think of a demon is as a model of sequential repeatable measurement "crafted" onto a model of (sequential) computation. The Turing machine and the von Neumann model are the two most immediate choices though, as we have seen, neither of these is "reversibility conscious". This

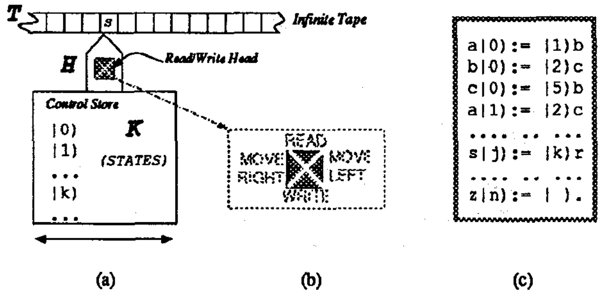

Figure 3: The Turing machine: (a) Hardware components, (b) Instruction Set and (c) "Program".

is not a fatal problem, though, because we now have well defined strategies to convert logically irreversible models into reversible ones. Moreover and as we have seen in the previous section the strategy to symmetrize individual measurements is s similar to the technique used to make computations locally reversible. This fact suggest that it is not an idle enterprise to put together a consistent formalism for irreversible demons as these can systematically be replaced with reversible surrogates through an effective prescription. The Turing machine assembly depicted in (Fig. 3) includes a central unit or *finite state control device* `K` with states $|\mathtt{i}), |\mathtt{j}), |\mathtt{k}), \ldots$, an attached a *read–write head* `H` and a recording *tape* `t` handling both the input/output and memory functions. The instruction set is composed of four instructions: read,write, move left and move right. The program D_T defining a given Turing machine T is a list of *n-tuples* describing each possible transition effected by the machine upon reading specific input symbols. For reasons which will become clear in the following we choose to represent them as *quartets* in the form:

$$D_T \equiv [a|0) := |2)b;\ b|2) := c(3|;\ \ldots;\ z|n) := |\ )]$$

where the *halt state* $|\ )$ — which need not belong to `K` — marks the end of the computation. Each of the quartets (e.g. $a|\mathtt{i}) := |\mathtt{j})b$ is interpreted to represent a statement of the form: *'when the machine reads a symbol a and is in state $|\mathtt{i})$ it makes an internal transition to state to $|\mathtt{j})$ writes the symbol b on the tape and moves to the right'*. The quartet $a|\mathtt{i}) := b(\mathtt{j}|$ would mean that the machine moves *to the left* (i.e. changes direction of motion overwriting the original input symbol a). It should be clear that this list of statements (suitably separated by semicolons as in (1) above) is itself a string which may be aptly called *a description* of the Turing machine T. The transitions encoded in the quartets are executed one at each step of the machine's evolution. In other words, the Turing machine implements a *sequential architecture*, where the G is a linear map. D_T does not, however, establish the the order in which the quartets are executed (the "Turing programing language" is non–procedural). In a deterministic machine that order depends on the initial state and initial symbol and different initial conditions will in general produce different computations. Some initial conditions may lead to indefinitely looping or non–halting behavior.

The overall (macro)*state* of T at any step t of its evolution may be completely characterized by an aggregate state vector $|\mathbf{k}, \boldsymbol{s}, [\mathtt{b}]_s\}$ composed by the current control state $|\mathbf{k})$, the currently scanned symbol $\boldsymbol{s}$ and the set of string blocks $[\mathtt{b}]_s$ remaining on the tape. The whole evolution of a Turing machine T under input of a tape $[\mathtt{I}]$ can thus be represented by $[\mathtt{I}]|\mathbf{k}, \boldsymbol{s}, I\} ::= | \;,\; , \mathtt{0}\}[\mathtt{0}]$. Turing showed that there are certain machines U with the property that, upon input of an encoded description $\mathtt{D}_T$ of another Turing machine T, U will, after a fixed transient number of steps, simulate the exact behavior of T and produce the same output after reading the same input string $[\mathtt{I}]$. This transformation encapsulates the notion of a Universal machine identifying the states:

$$U|\mathbf{k}, \boldsymbol{s}, \mathtt{C}, \mathtt{D}_T, \mathtt{I}\} \equiv V|\mathbf{j}, \boldsymbol{r}, \mathtt{I}\} \qquad (11)$$

or, in terms of the whole U evolution we can write:

$$[\mathtt{C}, \mathtt{D}_T, \mathtt{I}]|\mathbf{k}, \boldsymbol{i}, [\mathtt{b}]_i\} ::= |\mathtt{j}, \boldsymbol{r}, \; \}[\quad] \qquad (12)$$

In other words, a **universal** Turing machine (UTM) is a device whose behavior remains particularly insensitive to initial data. Its dynamics is no more than an interpretation of a machine suitably described to it and the transient block corresponds $[\mathtt{C}]$ to a decoding program for the initial conditions of the interpreted machine. The extended state description in (11) can be taken as *a complete encapsulation of all possible state labelings.*

The qualitative computational capabilities and limits of a UTM are summarized respectively by the Church–Turing Thesis and the Halting Theorem. These results give a definite meaning to the mathematical concept of Universality but they do not fully provide any quantitative criterion for deciding when a given Turing machine can be made universal. To avoid one more ambiguous usage of the loaded word "complexity", I will call the degree of Universal proclivity associated with a given machine its *versatility* Ξ ($\Xi = 1$ for universal machines and $\Xi < 1$ for non–universal).

The versatility of a TM is presumably a function of the 3 integers characterizing its resource space namely, n, the number of states, m the number of symbols in the alphabet and s the number of steps (quartets) in the program. The minimal amount of target resources (states and symbols) required for a Universal transformation are not known. Shannon[31] (1956) has shown that given a UTM with (n, m, s) it is possible to systematically build from it another UTM with only 2 states plus an alphabet of $m' = 4m + n$ symbols and a program of at least $s' = 2(n+1)s$ steps. He also pointed out that is is possible to implement a symbol reduction to a binary alphabet on any UTM by a corresponding increase in the number of states speculating that the product $n \times m$ is a 1st order measure of the versatility.

Another well known universality–preserving construction is the one which leads from any machine universal machine $U_{(n,m,s)}$ to its Bennett's ***reversible*** three–tape universal machine equivalent R_T with $n'' = 2n + 2s + 4$ states, $m'' = m, s+1, m$ symbols (on each tape) and $s'' = 4s + 2m + 3$ program steps. In section 4 these two procedures will be used in tandem to obtain a candidate Universal Quantum Demon with a small state space.

A few other items of the Turing machine folklore are relevant to the AP enterprise. Though Turing machines are usually associated with symbol processing or to the theory of recursive functions of the integers (which they simulate) there are numerous extensions of this theory which cover practically all computational tasks and number fields (including stochastic and parallel algorithms and functions over real and complex domains). Also equivalence theorems between other models and Turing machines are likely to form a more complete set than equivalence results to any other model. Moreover any of the obvious extensions of the Turing model (several tapes, several heads) have been shown *not* to lead to any increase in versatility or indeed to any computational novelty.

Finally *rules of composition* of TMs are well known and studied. There are two main TM "products": *sequential*, where the same shape is shared between two machines and *parallel* where the two machines read, write and move different tapes but their state space is the Cartesian product of the two spaces. The latter is particularly relevant for quantum demons as will be seen in the following, though composition, in the classical TM context, is known not to lead to any increase in versatility.

3.2 Models of measurement.

Unlike computation measurement is a purely interactive concept: its meaning is intrinsically associated with the coupling of a physical system (the measurement target) S, and another fiducial system called the measuring instrument, I. Assessing the notion of self–measurement, introduced by Albert[2] is which the two systems coincide is certainly one of in very specific circumstances as the interaction between I and S is normally a complex reactive process. Another distinction comes from the fact that while a computation is a process extended in time (and in space in the case of cellular automata) a measurement is often conceived as a well localized *event* in space and time. This is the standard case in the von Neumann model (vNM) where one implements the measurement interaction in the *impulse* approximation[35].

Given that the vNM by no means provides a general characterization of measurement[3] one may justifiably ask what model can claim such generality? **AP** suggests that the simplest universal model of measurement is already a computing device. We will show in the following how this suggestion comes about. As we have seen in section 2 the measurement map can be de-constructed as a sequence of three distinct series of laboratory manipulations: (1) the *preparation* of the initial joint state of system and instrument, followed by (2) a time limited *evolution* where the two interact ending in (3) the *detection* by I of the value of a at the end of the interaction. Stages (1) and (3) both involve filtrations or selections of states while stage (2) is fully governed by the Schrödinger equation.

Any quantum mechanical state corresponds to a ray in Hilbert space which can be identified with the laboratory procedures used to realize it[34]. The simpler preparations produce pure (tensor) product states of S and I proportional to $|\psi\rangle_S|\Phi\rangle_I$ and are associated with a set of projection operators (filters) $\{P^{\Phi}_{A'} = |\Phi_{A'}\rangle\langle\Phi_{A'}|\}$ each one associated with a value A' of the preparation variable A. These operators characterize the instrument completely and satisfy both *completeness* and *orthogonality*:

$$\sum_{K'} P^{\Psi}_{A'} = 1; \qquad P^{\Phi}_{A'}P^{\Psi}_{A''} = \delta_{A',A''}P^{\Phi}_{A'} \tag{13}$$

We should, however, beware that quantum mechanics allows for the preparation of *entangled states*, that is, superpositions of states of two systems which do not decompose into products of individual states i.e. $|\psi_1\rangle_S|\Phi_2\rangle_I + |\psi_2\rangle_S|\Phi_1\rangle_I$. This is one of the several reasons for enlarging the set of elementary preparations beyond the set (13). The other complications concern the non-ideal nature of instruments and the need to account for statistical mixtures of states. The most general "preparation set" seems to be associated with so-called *Positive Operator Valued Measures*[10, 19] (POVM). It can be shown[23] that these generalization does not invalidate the schematic description of preparation we will be using, namely that of an operation P which results in an interactive (non-analyzed) state:

$$P^{\phi}_{S}|\Phi\rangle_I \mapsto |\Psi_i\rangle_{S,I} \tag{14}$$

The middle stage evolution couples the physical variable of S to be measured, a, to a corresponding variable, A, of the instrument I_A. A simple choice (if a is canonical) for the Hamiltonian driving this evolution is the linear interaction $H_{int} = -g(t)aA$ with $g(t)$ a positive coupling constant. The canonical conjugate to A, Π_A becomes the "pointer observable" for the measurement and is given by Hamilton's equation $\dot{\Pi}_A = -\partial H/\partial A = g(t)a$ or jointly by the commutation equations $[\Pi_A, A] = i\hbar$ and the Heisenberg equation of motion for the instrument:$-i\hbar dA/dt = [A, H]$. Its variation due to the interaction is proportional to a. The evolution may thus be described also as a transition:

$$E^a_A \equiv U(t)|\Psi_i\rangle_{S,I} \mapsto |\psi(t)\rangle_S|\Phi(t)\rangle_I \tag{15}$$

with $U(t) = \exp\{-i\int_0^t H_{int}(\tau)d\tau\} = \exp(iGaA)$ and $G(t) = \int_0^t g(\tau)d\tau$. Assuming that a is a discrete nondegenrate quantity associated with an operator $\hat{a}$ with spectrum $\hat{a}|a_n\rangle = a_n|a_n\rangle$ we can expand the system's initial state as in $|\psi_i\rangle = \sum_n c_n|a_n\rangle$ so that the final

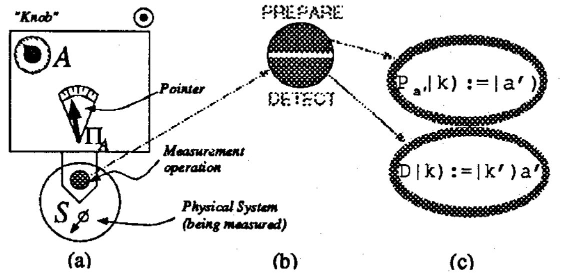

Figure 4: Von Neumann Measurement Model: (a) Assembly, (b) Instrument Set and (c) Elementary Operations

product state has the decomposition:

$$\sum_n \exp(iGa_nA)c_n|a_n\rangle|\Phi_i\rangle_I = |\Pi_A - Ga_n\rangle|a_n\rangle$$

as the exponential factor is a translation operator for the instrument in its canonical conjugate representation. The eigenstates $|\Phi_i\rangle$ of the measuring device are often chosen to be *gaussian* wave packets and, because their relative sharpness determines the accuracy of the measurement, their spread should be significantly smaller than the separation of the eigenstates $|a_n\rangle$ (i.e. G should be large). E^a_A evolves the initial state of the instrument to a superposition of these gaussian packets, each centered around one of the eigenvalues of the system;

The final detection step *collapses* this superposition leaving the instrument in one of the gaussians states. The outcome of the measurement can be read off the Π_A scale: $a' \sim \Delta\Pi_A = \Pi^f_A - \Pi^i_A$ so this detection can be viewed as a *conjugate* transition in the instrument's state:

$$D^{\Psi}_A|\Pi_i\rangle_I \mapsto |\Pi'_A\rangle_I|a_f\rangle_S \tag{16}$$

where

$$|\Pi'_A\rangle \equiv |\Pi_A - Ga_f\rangle = |\Phi_f\rangle\langle\Phi_f|exp(iGa_fA)|\Phi_i\rangle$$

It is this relativisation of the measurement process to a series of instrument state transitions which strongly suggests a computational analogy. To complete it we need to introduce a protocol to describe repeated measurements in discrete time steps.

3.3 Measurement symbols

The transitions (14),(15) and (16) may be encapsulated in the following definition of a ***measurement symbol***[30] $\boldsymbol{a}$ associated with the target variable a:

$$\boldsymbol{a} \equiv \sum_{\phi'}\langle\psi_f|X_A|\phi'\rangle\langle\phi'|\psi_i\rangle$$

Where X stands for any of the above measurement transitions. Detection symbols, for example, are associated with projections over final states and consequently with the eigenvalues of the corresponding preparation symbols:

$$a' = \langle\psi_f|a'\rangle\langle a'|\psi_i\rangle = \langle\psi_f|P_{A'}|\psi_i\rangle$$

The simplest detections have binary outcomes, indicating simply the presence or absence of the outcome value within a given test set as in:

$$\Delta_{\tilde{A}}|\Psi_i\rangle = \begin{cases} |1\rangle|a'\rangle, & \text{if } a' \in \tilde{A} \\ 0, & \text{otherwise} \end{cases}$$

More importantly one can devise construction protocols for concatenating symbols into strings or ***experiments*** and thus to build *manuals* for repeatable discrete measurement. The resulting sets are similar to formal languages, though restricted by the very specific limitations imposed by commutation relations. For particular measurement environments it is relatively easy to find a set of *initial* measurements from which to build all the possible laboratorial situations where the finite set of target quantities can be measured. We have already seen that the preparations and detections are associated mostly with projections (or more realistically with POVMs).

The coding of evolution symbols are considerably more involved specially because quantum evolutions do not occur under the control of the instrument but under that of the equations of motion. Thus evolutions cannot be included as part of the "instruction set" of the instrument. They can nevertheless be expressed in the language of projections through finite difference discretization along the lines of equation (1) and the replacement of the continuous Hamiltonean H_{int} with a step–wise version[16]:

$$\mathtt{H}_1 = \mathtt{g}|\mathtt{t}-1\rangle\Upsilon_{(a,A)}\langle\mathtt{t}| + |\mathtt{t}\rangle\Upsilon^{\dagger}_{(a,A)}\langle\mathtt{t}-1| \tag{17}$$

where Υ is a scalar operator describing the interaction. For the linear case it is simply a GaA times a θ–function which is one between $\mathtt{t-1}$ and $\mathtt{t}$ and zero elsewhere. In this case the operators Υ are just simple creation and or annihilation operators for the fiducial final states. More generally they can be linear functions expressible in terms of the initial symbols $\{a, a', a'', b, b', \cdots\}$ and also of the external parameter settings ("knobs") of the instrument, that is, the values of the pointer observables $\{A, B, C, \cdots\}$ In the discrete realm evolutions are reducible to polynomial functions of projections[?] and other more trivial operators.

The above analysis of the vNM allows us to picture a measurement instrument as in fig. 4, that is, as a device which also has a basic physical assembly and interacts not with a tape but with an external physical system and is equipped with an *operation set* including preparations and detections encodable in symbolic measurement language. When these measurements are repeatable we can draw an extensive analogy between an automated instrument or sequence of instruments and a computer.

3.4 Orders of Demons

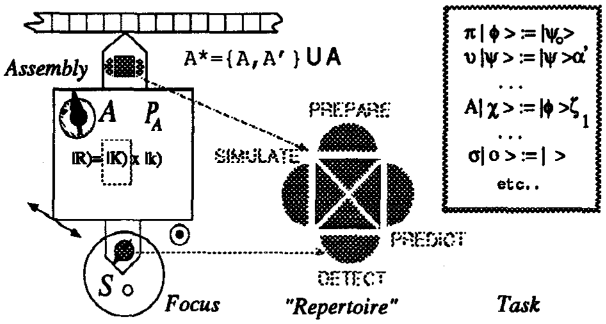

Figure 5: Schematics of a Demon.

We have only to superpose figures 3 and 4 to assemble our first generic "demon" (Fig. 5): a Turing machine which can be programmed to execute measurements. The assembly includes a tape and a moving read-write head as well as one or more primitive instrument heads while the states space has become a *tensor product* of the state spaces of both instrument and system understood as their associated Hilbert space when they are quantum systems.

The alphabet now contains both measurement and value symbols with lead to measurements being executed and more conventional logical or numerical symbols targeted by the instrument heads. As for the instruction set it is also generalized to a *"repertoire"* that includes not only elementary preparations, evolutions and detections but also two "mixed" or second order manipulations which we call *predictions, retrodictions* and *simulations*. These correspond to computations bearing on the same set of physical quantities being measured. They have the same representational structure as the first order operations (14), (15) and (16) but activate only the tape sector of the demon:

$$Q_A[\mathrm{D}]|\Phi_{A'}\rangle \mapsto |\Phi'_A\rangle|\Pi'_A\rangle \tag{18}$$

$$R_A[\mathrm{P}]|\Pi'_A\rangle \mapsto |\Phi_i\rangle|\psi_i\rangle \tag{19}$$

$$S_{a,A}[\mathrm{E}]|\Phi_i\rangle \mapsto |\Psi_f\rangle \tag{19}$$

These are second order in the sense that they have 1st order demons (more precisely, their coded descriptions) as 'arguments'. A predictor attempts to determine the results of pointed to by an encoded detector, for example. Higher order demons are also possible and one last variety is particularly needed to cap our inventory. These we call *comparators* as they purely evaluate the accuracy δ of, say, predictions for the outcomes of a given detection compared with the corresponding outcomes of the detections:

$$C_A[Q, D]|\Phi_{D(A)}\rangle \mapsto |\Phi''_A\Pi'_A\rangle\delta(A'', A') \tag{20}$$

Prediction and simulation algorithms are not precisely those provided by classical or quantum mechanics but

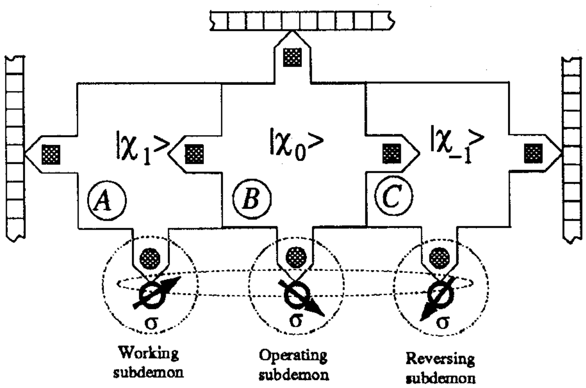

Figure 6: Simple three part reversible Demon for spin 1/2 measurements.

most likely include pragmatic devices such as numeric approximants and/or Monte Carlo or Molecular Dynamics techniques which provide finitary implementations of physical laws or tentative models. In the best spirit of Artificiality we would like to allow predictors which give 'false' or incomplete predictions given that quantum mechanical models do not typically predict the outcomes of individual measurements but only their statistical distribution.

We would like to accommodate among the predictive demons models such as the Bell's deterministic hidden–variable algorithm[4] which correctly predicts the value of measurements carried out on a single spin 1/2 degree of freedom. The extended Bell's demon is known to fail as a deterministic predictor for systems of two or more spins — an effect of the entanglement of the product states. In the **AP** context we would like to understand the nature of this failure in computational terms and find out whether an effective Bell demon would be non–halting, for example.

4 A candidate Quantum Demon

One of the first questions posed by **AP** is whether one can give meaning to a notion of Universal Demon. In other words: given a finite set of physical variables can we build a demon which is a universal computer and is also capable of conducting all strings of experiments spanned by its repertoire of measurement operations? The tentative answer to this question is that a demon which can be manipulated to implement all the seven type of operations described above on a given set of physical quantities captures the essence of universality in the AP context in that it can be proven that it will simulate any other demon. The proof of this statement is quite involved and tedious but the content of the statement can be made intuitively persuasive through an eloquent example. Fig.6 displays schematically the what I believe is the simplest quantum demon with a chance of deploying universal behavior in this enlarged sense. It is composed of three irreversible blocks identical to the standard fig.5 demon each with two dimensional state–spaces, represented by three spin magnetic fields whose axial direction can be manipulated by three independent knob parameters $\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}$. The interaction Hamiltonian for parallel simultaneous measurements is:

$$H = \hbar\omega(\hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{A}})(\hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{A}})(\hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{C}})$$

It can be obtained formally by applying to any presumed but unknown universal demon, Shannon's procedure to reduce the state space to two dimensions followed by Bennett's procedure to make the demon reversible. The resulting 3 sub-demons perform the three sequential functions of the Bennett machine in a somewhat more complete form as the "operating" or history sub-demon is allowed to detect the state of the other two. It can be proven[23] that such device is reversible in both a global context of an extended experiment but also locally for each measurement in the sense implied in the ABL model sketch previously. It is also particularly easy to build a set of initial recursive spin measurements perform-able by this demon, namely:

(i) The **null** measurement associated with the operator:

$$\sigma_o|\quad\rangle := 0; \sigma_o \equiv (\sigma_x^o - i\sigma_y^o)/\sqrt{N}$$

(ii) The **successor** measurement:

$$\sigma^+|m\rangle := |m+1\rangle; \sigma_m^+ \equiv |m+1\rangle\langle m| = (\sigma_x^m + i\sigma_y^m)/\sqrt{N}$$

(iii) The **projector** measurement:$\pi_{m'} = |m'\rangle\langle m'|$ with, for the spin case:

$$\pi_m^+ = \frac{1}{2}(1+\sigma_z^m) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \pi_m^- = \frac{1}{2}(1-\sigma_z^m) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

This quantum demon was built however for the specific task of in the foundations of Quantum Mechanics: that of analyzing in a systematic fashion the relations between the Bell's inequalities and the more recent results of Greenberg,Horn, Shimony and Zeilinger[18] and of Mermin and Peres, both pointing to the discrepancies of classical and quantum predictions. This inquiry is still in its early stages but serves as a example of the kind of tasks in which one may use the wits of a demon with some advantage. The suggestion that AP may shed some light in this problem comes from two apparently contradictory observations. In the wake of the GHSZ argument Zeilinger[36] has noted that "entanglement is itself an entangled property" in the sense that it may be propagated or reduce by carefully selected successive measurements. His conclusion points to a *recursivity of entanglement* but Peres[29] points out that GHSZ may be interpreted as a limitation in the recursive definition of elements of physical reality!

Our analysis based on simulations of the triadic demon described above, points instead to a trade off between entanglement and recursivity as far as the effective operation of demons is concerned. Though the full details are too involved for this introduction I would like to complete the "pyramid of recursivity" for generic measurement operations carried out by our demon, to illustrate the nature of the problem. The functions involved in Bell type arguments are the expectation values $\langle A \rangle_\chi = \langle \chi | A | \chi \rangle$ having the three target spins as arguments. Their structuring operations are:

Composition:

$$\langle A \rangle_\chi = \langle \langle B \rangle_\chi, C \rangle_\Psi$$

Recursion:

$$\langle A, 0 \rangle_\psi = \langle C \rangle_\chi$$

$$\langle A, \sigma^+ B \rangle_\psi = \langle A, B, \langle A, B \rangle_\psi \rangle_\Xi \qquad (22)$$

Unbounded minimization:

$$\langle B \rangle_\chi = \overline{\min}_\chi \langle \langle A \rangle_\chi, A \rangle_\psi =$$

$$\begin{cases} \min_{\chi'} \langle \langle A \rangle_\chi, A \rangle_\psi = 0, & \text{if } \chi \text{ exists} \\ 0, & \text{otherwise.} \end{cases}$$

with χ labelling single spin, ψ two spin and Ξ three spin states and the A, B, C. As it turns out these assignments which span all the finite operations carried out by triadic demons offer a promising framework for reconciling entanglement and recursion in the restricted context of few spin experiments.

5 Farther Horizons

Artificial Physics presents an alternative road for the symbiosis of Physics and Computation, somewhat less committed than most others to subsume one of these fields to the other but intent, instead in finding out what each of its individual approaches may bring to the other one. The language is new and still being articulated and sharpened but new problems and applications already loom in the horizons of this new field. Perhaps the most pressing, most daunting and also most exciting is the empirical construction of demons. The triad demon for example resumes a series of operations which have been considered for dedicated experimental verification. A design for a superconducting interferometer which would act as an analog simulator for this demon is currently under study. Other inquiries from upcoming Artiphysicists are forthcoming...

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