Quantum Mechanical Neural Networks: An Isoperimetric Extremization

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Abstract -This paper introduces the idea of a new and novel approach to Neural Networks. Considering the Hopfield Model as a prototype, the apparatus of Quantum Mechanics is used to quantize the resultant weight field. A conjecture is made about the correspondence between Hopfield attractors and eigenvalues. Directions for continued work are given.

Introduction

Many popular authors of scientific speculation pose a form of holographic mind. What can this mean from the physics of computation viewpoint? Is the normal representation, based on Lorentz invariance, going to remain valid for the discussion of domains that overlap thought and the physical worlds? Can the power of information representations that has been achieved in physics be applied to the modern tasks of every day life? I am going to claim as a generalized correspondence principle that physics is even more general than we as physicists can imagine. This leads us to consider the structural forms of Quantum Mechanics (QM) outside of their usual domains of definition.

The central proposition of this paper is: *invariant embedding is the proper framework for our theoretical extension of Quantum Mechanics into the domain of model-free computation with Neural Networks.* We make an observation about the general Hopfield model, examine the original writings of Erwin Schrodinger, and *viola* - Quantum Mechanical Neural Networks pop out. Using the theory of invariant embedding we identify the new objects based on observational data about the Classical Energy Neural Networks (CENN).

The remainder of the paper highlights these major points, and identifies area to probe deeper. The major points are:

1. The energy formulation of Neural Networks
2. Erwin Schrodinger's original three (3) papers
3. The Isoperimetric extremization technique
4. Analysis of the results and identification of new objects.

Classical Energy Type Neural Networks: *a la* Hopfield's Model

The so-called Hopfield Model (HM) is a Steepest Descent Optimizer useful for the solution of optimization problems. The traditional approach, looking at the time evolution of the energy-like function the CENN is to minimize, corresponds to the classical phase space development. The dynamics of the HM involve looking at the time derivative of the generalized energy terms. Hopfield adds a potential looking term that has a sigmoidal function behavior. Pattern matching occurs when the input vector is appropriately close, in some defined metric sense, to one of the patterns for the family of attractors of the model. Partial truth for an input vector implies a closeness to the corresponding attractor .

This is very much like the state of classical physics in 1926, before Schrodinger. The world looked at the Time Reduced Hamilton-Jacobi equation for a single particle state evolution. We shall take the same road - in an invariant embedding sort of way.

Classical QM *a la* Schrodinger

Schrodinger's first paper considers the Time Reduced Hamilton-Jacobi equation for a point particle of mass, m, moving under the influence of an arbitrary force field created by the generalized potential, V, with, E, identified as the total energy of the particle:

$$\frac{1}{2m}\left[\left(\frac{\partial S^*}{\partial x}\right)^2+\left(\frac{\partial S^*}{\partial y}\right)^2+\left(\frac{\partial S^*}{\partial z}\right)^2 \right]+V(x,y,z)-E=0$$

With the change of variable $S^* = k\log\Psi, k$, a constant (TBD), we have after multiplication by, Ψ^2, :

$$\frac{k^2}{2m}\left[\left(\frac{\partial \Psi}{\partial x}\right)^2+\left(\frac{\partial \Psi}{\partial y}\right)^2+\left(\frac{\partial \Psi}{\partial z}\right)^2 \right]+(V-E)\Psi^2=0$$

Schrodinger then considered the volume integral of this equation with the question: What differential equation must the function , Ψ ,satisfy if the integral is to be an extremum with respect to twice differentiable functions, Ψ ,which vanish at infinity such that the integral exists? The answer is to substitute the integrand into an Euler-Lagrange operator style equation:

$$[\ \frac{\partial}{\partial\Psi} - \frac{\partial}{\partial x}(\frac{\partial}{\partial\Psi_x}) - \frac{\partial}{\partial y}(\frac{\partial}{\partial\Psi_y}) - \frac{\partial}{\partial z}(\frac{\partial}{\partial\Psi_z})\ ]f = 0$$

to obtain:

$$-\frac{k^2}{2m}(\Psi_{xx} + \Psi_{yy} + \Psi_{zz}) + (V - E)\Psi = 0$$

as our partial differential equation which must be satisfied. We recognize this as the Time-Independent Schrodinger Equation for one particle. (The extension to many particles is straight forward.)

In the next paper, Schrodinger extremizes the integrand, leaving the potential energy term alone. The extremization is carried out with respect to functions, Ψ, which satisfy the isoperimetricly normalized condition:

$$\int \Psi^2 d\tau - 1 = 0$$

We are thus lead to identify the energy of the system with the undetermined Lagrange multiplier. "Thus the Schrodinger eigenvalue-eigenfunction problem is equivalent to the above isoperimetric problem..." We will ask the same type of question.

The Imaginative Leap to Quantum Mechanical Neural Networks

OK - we have an energy-like function *via* Hopfield, and a quantization technique *via* Schrodinger. We have an N-dimensional optimization process that bounds the family of attractors. When we quantize a system in quantum mechanics, and we quantize the matter field, what we do is make a transformation to the tangent space: so now we have tangent planes that we talk about. This is the Hamilton-Jacobi Action, as above. The questions is: *if we were to blindly look at the weights that are in a mass-type position in the Hopfield functional, and say if the mass in a Schrodinger equation could then correspond to some kind of weight quantization, what would the eigenvalues be, and are they close to the attractors in the Hopfield classical network?* What role do the eigenfunctions play? We have a lot of operator theory and elegant representation theory to consider. To me, *computation is the representation theory, or maybe it's true that the representation theory is the computation.* But we have all of this work that's been done. Is it applicable to looking at the questions of optimization that neural networks are doing?

The question for this audience is: "Under what conditions are these formalistic manipulations valid?" We know from invariant embedding theory that "like goes over to like" and we can identify the rest of the new objects in the theory. We can symmetrize the energy term *via* group theoretical arguments like we use in molecular spectroscopy. We have generated the wave function and we need to investigate the operators associated with the standard QM theory. We have many large hammers in the shop called Physics that we might use on this problem.

The road left to you, the reader

We end up with a quantized weight field (instead of the matter field being quantized). Do these quantized weights lead us to the holograhic memory models? How do we interpret the new eigenvalues? Do we have to suffer the "philosophy rap" over these eigenfunctions too? Will there become a Dallas interpretation of QMNN's? There is much to be looked at. An understatement. It all remains to be investigated. Let's remember to acknowledge this conference... this environment. Thank you.

Bibliography

1. Abraham, Ralf and Jerrold E. Marsden. 1985. *Foundations Of Mechanics.* Benjamin/Cummings Publishing Company, Reading, MA.
2. Bellman, R. and Wing, G.M. 1975. *An Introduction to Invariant Imbeddimg.* Wiley-Interscience Series, New York, NY.
3. Goldstein, H. 1950. *Classical Mechanics.* Addison-Wesley, Reading, MA.
4. Papadakis, Ioannis N. M. 1991. *On the Stability and Robustness of the Hopfield Model.* Intelligent Engineering Systems through Artificial Neural Networks, ANNIE '91. ASME Press.
5. Weinstock, Robert. 1952. *Calculus Of Variations - With Applications to Physics and Engineering.* McGraw-Hill Book Company. New York.
6. Wells, Dare. 1967. *Theory and Problems of Lagrangian Dynamics.* Schaum Publishing Company, McGraw-Hill Book Company. New York.