Entropy Flow in a Mesoscopic Conductor and the Entropy of Erasure

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Abstract

The 'discretized' thermopower of a quantum point contact can be understood in terms of the entropy carried by an electron at the Fermi energy across the conductor. This electron acquires an entropy $k_B \ln 2$ by a process analogous to the entropy generated by erasing a single bit of information. Inserting a scattering obstacle into the point contact generates additional entropy. When the obstacle has a reflection coefficient $R = 1/2$, it generates an additional 'erasure' entropy of $k_B \ln 2$ per injected electron. Attaching superconducting leads to the point contact eliminates the entropy and heat current flow, showing the essential role of the measurement leads in generating an entropy current.

1 Introduction and Motivation

The thermoelectric properties of a conductor result from a balance of the particle and entropy currents inside the device. In the same way that the flow of charged particles determines the electrical currents in a conductor, the entropy current flow in a physical system determines the heat current flow. Temperature measurement requires reliably sampling the entropy currents at a point, and, like a voltage measurement, can be strongly affected by the choice of measuring 'leads' attached to the sample.

But understanding the thermoelectric properties of conductors is not the only motivation for studying the flow of entropy currents. Information flow is intimately related to the entropy of arranging individual 'bits', where these bits might consist of quantum spins, trapped electrons, or electron wavepackets moving through a conductor. In this paper we compare the particle, entropy, and information currents flowing through a quantum point contact.

2 Physical Entropy Flow

The quantum point contact [1] consists of two thermodynamic reservoirs (of electrochemical potential μ and temperature T) separated by a small orifice. Electrical transport through a quantum point contact can be viewed in an 'electron wavepacket' picture [2]. The emitter contact launches successive electronic wavepackets, which are then received by the collector. The energy bandwidth $\Delta E = \mu_1 - \mu_2$ of such a conductor is simply the applied voltage eV, so that successive electrons follow each other every h/eV seconds [3] as shown in Fig. 1.

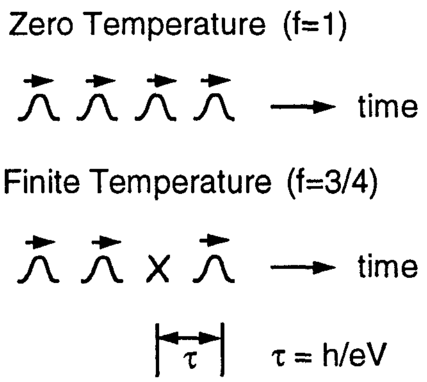

Figure 1: Electron wavepackets flowing out of a thermodynamic reservoir are interspersed with the absence of such packets (X's) at finite temperature. An entropy current and a corresponding heat current therefore flow out of the reservoir when $T \neq 0$.

At finite temperature the probability of emitting a wavepacket is governed by the Fermi occupation factor f. There are many different ways to arrange the emitted wavepackets and vacancies while maintaining the average occupation f. Therefore, an entropy current will flow out of the emitter at finite temperature, due to the temporal irregularity of injected wavepackets (noise). The emitter entropy $H(X)$ is simply the logarithm of the number of ways to arrange the packets, namely

$$H(X) = -k_B \left[f \ln f + (1-f) \ln(1-f) \right], \quad (1)$$

shown in Fig. 2. When $f = 1/2$, so that the electron has no greater likelihood of being either present or absent, the entropy $H(X)$ is a maximum. The maximum $H(X) = k_B \ln 2$ corresponds to the erasure entropy [4].

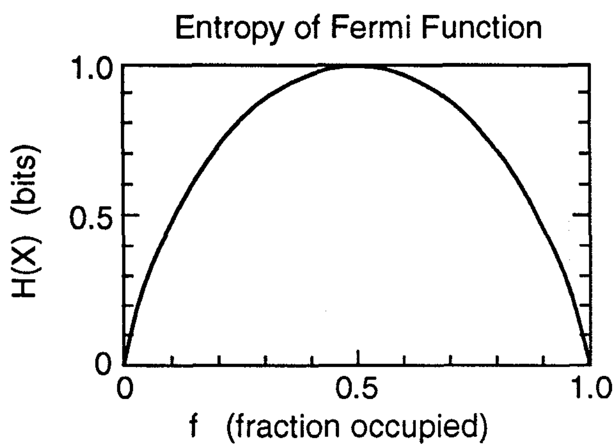

Figure 2: Informational and physical entropy $H(X)$ of the Fermi occupation factor f.

The rate of entropy flow I_S out of the emitter in a small energy interval dE near energy E is now simply $I_S = H(X)\ dE/h$. Thus, the physical entropy flow written down in Ref. [5] can be understood using the wavepacket picture of Ref. [2].

3 'Discretized' Thermopower

Whenever the orifice of the point contact widens to admit a new electron wavelength, where the number of such open conducting channels is N_c, its conductance G increases by $2e^2/h$ as shown in Fig. 3. At the transition between two conductance plateaus, the thermopower S shows a peak [5]-[7].

To understand the peaked structure of the thermopower, it is necessary to account for the entropy flow due to wavepackets emitted from the collector. The entropy flow out of the collector opposes that from the emitter, so that the net entropy flow is zero when both wavepacket streams flow unimpeded. But a small voltage bias applied across the conductor raises the emitter Fermi energy is slightly above that of the collector. The entropy current out of the collector can then be partially blocked near the transition between two conductance plateaus. A net entropy current can thus flow across the conductor, manifest as a peak in the thermopower between conductance plateaus.

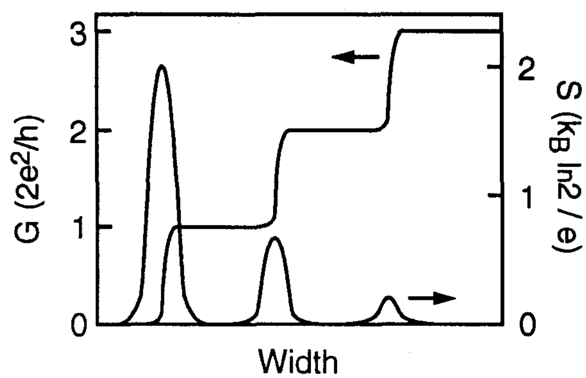

Figure 3: Conductance G and thermopower S of a quantum point contact.

Using the analysis of Ref. [5], we can obtain the net electrical current I and net heat current I_Q. On a conductance plateau we have

$$I = \frac{2e}{h} N_c(\Delta\mu) + 0 \cdot (\Delta T), \quad (2)$$

and

$$I_Q = 0 \cdot (\Delta\mu) + \frac{2}{h} A N_c (2k_B T)(k_B \Delta T). \quad (3)$$

Here $A = \int_{-\infty}^{\infty} x^2 \text{sech}^2 x \ dx$, a number of order one. In the middle of a conductance step we find

$$I = \frac{2e}{h}(N_c + 0.5)(\Delta\mu) + \frac{2e}{h}(k_B \ln 2)(\Delta T), \quad (4)$$

and

$$I_Q = \frac{2}{h}(k_B T \ln 2)(\Delta\mu) + \frac{2}{h} A (N_c + 0.5)(2k_B T)(k_B \Delta T). \quad (5)$$

The first term in Eq. (5) says that, in the middle of a conductance step, only the 'half electron' in the partially open channel at the Fermi energy carries the 'erasure' entropy $k_B \ln 2$. The remaining entropy flow in the fully open channels below the Fermi energy is compensated by an opposing flow from the collector, as discussed above, and does not contribute to the net heat current flow. In contrast, all the $N_c + 0.5$ open channels contribute the net electrical current flow. Therefore, the increase in electrical energy $e(N_c + 0.5)(\Delta V)$ compensates the increase in entropic energy $k_B \ln 2(\Delta T)$, leading to a maximum thermopower [6]

$$S \equiv \left.\frac{\Delta V}{\Delta T}\right|_{I=0} = -\frac{k_B \ln 2}{e(N_c + 0.5)}, \tag{6}$$

where $N_c = 0, 1, 2, \ldots$. This thermopower applies when the conductor is open circuited, by taking $I = 0$ in Eq. 4. The second term in Eq. (4) can be interpreted as an entropy driven electrical current.

4 Entropy Generation by a Barrier

A barrier to particle transmission, having transmission coefficient $T = 1 - R$, will alter the entropy current flow. To illustrate this, we return to the simpler case where only one source emits wavepackets, shown in Fig. 4. The entropy in the incident wavepacket stream from the emitter is still $H(X)$. The entropy $H(Z)$ in the reflected wavepacket stream flowing back into the emitter is

$$H(Z) = -k_B \left[Rf \ln Rf + (1 - Rf)\ln(1 - Rf)\right]. \tag{7}$$

The entropy $H(Y)$ in the transmitted wavepacket stream flowing into the collector is

$$H(Y) - k_B \left[Tf \ln Tf + (1 - Tf)\ln(1 - Tf)\right]. \tag{8}$$

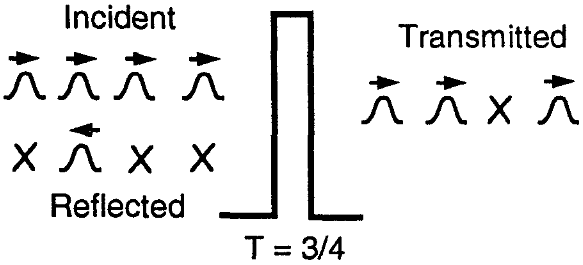

Figure 4: An outgoing entropy flux is generated by a partially reflecting barrier, even though no entropy flows into the barrier.

A key physical assumption in Ref. [5] is that one can completely neglect the correlations between wavepackets in the streams flowing into and out of a reservoir. This assumption seems to work well in describing experiments [7]. One can therefore add the entropy flows out of a reservoir, obtaining the net physical entropy current flow out of the emitter as

$$I_{Sr} = [H(X) - H(Z)]dE/h. \tag{9}$$

The physical entropy flowing away from the barrier on the left is

$$I_{Sl} = H(Y)dE/h. \tag{10}$$

Note that $I_{Sr} \neq I_{Sl}$.

When the incident wavepacket stream is filled, so that $f = 1$, the barrier acts as an entropy current source of magnitude $I_{Sr} = -I_{Sl} = -k_B(T \ln T + R \ln R)dE/h$. Consider the case where the transmission probability $T = 1/2$. The electron wavepacket is then incident on the barrier from a definite direction, but leaves in a random one. As a consequence, this partial reflection generates an 'erasure' entropy $k_B \ln 2$ per electron.

5 Informational Entropy Flow

In this section we again consider the simplified conductor of Fig. 4, where the emitter (sender, X) is launching wavepackets through a barrier into an empty collector (receiver, Y). The presence of an electron wavepacket can be viewed as a logical 'one', and its absence as a logical 'zero'. The machinery of information theory [8] can then be applied to compute the flow of the 'decrease of ignorance' across the conductor. Our problem here is equivalent to the standard 'Z-channel' problem in information theory. Note the information being transmitted concerns only the electron source itself, and not information about voltages applied to the conductor.

The rate I_I of mutual information flow [8] from emitter to collector is found

$$I_I = [H(X) - H(W)]dE/hk_B, \tag{11}$$

where the conditional entropy $H(W) = H(X|Y)$ is

$$-\frac{H(W)}{k_B} = (1-f)\ln\left(\frac{1-f}{1-Tf}\right) + Rf \ln\left(\frac{Rf}{1-Tf}\right). \tag{12}$$

To measure the entropy in bits we send $\ln \rightarrow \log_2$ and $k_B \rightarrow 1$ in all formulas.

For a ballistic channel, where $T = 1$, the physical entropy flow is equal to the informational entropy flow, $I_{Sr} = I_{Sl} = k_B I_I$. From the viewpoint of information theory, the quantized conductance of a point contact can thus be viewed as the maximum information transmission capacity I_I of a single quantum channel [9]. The maximum capacity of such a noiseless channel is simply the channel bandwidth $I_I = \Delta E/h$. However, for a partially transmiting channel, with

$T \neq 1$, none of the entropy flows are the same, so that $I_{Sr} \neq I_{Sl} \neq k_B I_I$. Thus, in the disordered channel, the physical entropy flow and informational entropy flows are not equal.

6 Demonology

To further understand the inequality of physical versus informational entropy flow, we introduce an ideal observer or 'demon'. Consider a finite automaton, the 'demon' shown in Fig. 5, who can non-destructively observe both the incident and transmitted wavepacket streams. After observing the incident and transmitted wavepacket for each time interval, the automaton writes either a one, a zero, or nothing into its memory. We will construct two different demons, such that the receiver can reconstruct the sender's pulse train by examining only the transmitted pulses and the demon's memory.

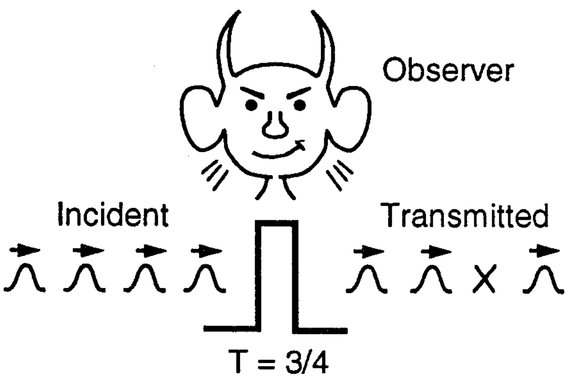

Figure 5: An automaton can observe both the incident and transmitted wavepacket streams.

Consider first a 'stupid' demon, who writes a zero in its memory if the incident and transmitted packets are the same, and a one if they are different. This 'stupid' demon is simply a NAND gate writing into a memory. When this demon erases its memory, it will generate the erasure entropy $H(Z)$. The physical entropy flow is the source entropy $H(X)$ minus the informational entropy $H(Z)$ of the 'stupid' demon Z.

Consider next a 'smart' demon, who writes nothing into its memory if the receiver gets a one. This is because, if the receiver gets a one, the sender must have sent a one. The 'smart' demon writes a zero in its memory if the receiver gets a 'correct' zero, and a one if the receiver gets an 'incorrect' zero. The 'smart' demon generates an entropy $H(W)$ when it erases its memory. The informational entropy flow is the source entropy $H(X)$ minus the informational entropy $H(W)$ of the 'smart' demon W.

The 'smart' demon W is a more complex automaton than the 'stupid' demon Z, but generates less entropy when its memory is reset. The 'stupid' demon Z writes into its memory on every pulse cycle, being a wasteful automaton, but the 'smart' demon W need only write for a fraction $1 - Tf$ of the total number of pulse cycles. Because $H(W) \leq H(Z)$, the informational entropy flow is larger than the physical entropy flow in our disordered open system.

7 Superconducting Point Contact

We now replace the normal electron reservoirs of the point contact with two superconducting reservoirs. Any normal electrons incident from the point contact on the superconducting reservoir will 'Andreev reflect' as a normal 'hole'. These incident electron and Andreev reflected hole wavepackets are depicted schematically in Fig. 6.

This Andreev reflection process is responsible for the poor thermal conductivity of superconductors, since it permits no electronic entropy to enter the superconductor. To compensate for the entropy current I_S attempting to flow into the superconductor (in the form of a conduction electron), the superconductor ejects an equal amount of entropy $-I_S$ flowing in the opposite direction (in the form of a 'hole'). Unless an electron tries to flow into the superconductor outside its energy gap, the superconductor does not permit its entropy to increase.

Despite being a poor thermal conductor, the superconductor-normal interface remains a good electrical conductor, since electrical currents are enhanced by Andreev reflection. The lattice of the superconductor can still carry heat, but the lattice contribution to the thermal conductivity is much smaller than the electronic one for normal metals at low temperature. The superconductor therefore appears as a relatively poor thermal conductor at low temperature.

Consider next adding a reflecting barrier inside the superconducting point contact [10]. In a naive viewpoint, electron scattering from the new obstacle inside the Josephson junction would produce an additional entropy proportional to $-k_B(T\ln T + R\ln R)$. Yet, due to the Andreev reflection mechanism, no entropy can flow into the superconducting leads. Therefore, the barrier cannot be producing any additional entropy current.

Attaching superconducting leads to the point contact has elimitated both the entropy production and

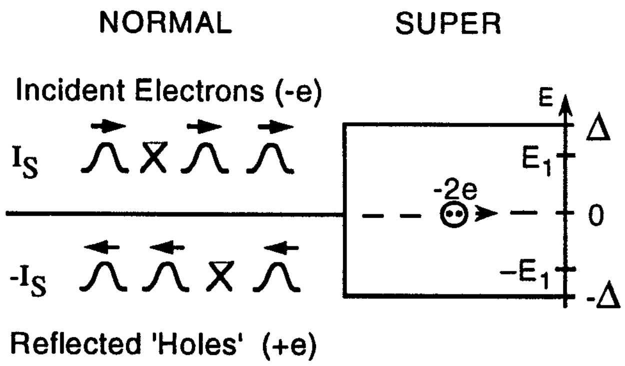

Figure 6: The Andreev reflection process permits electrical currents, but not entropy currents, to flow into a superconductor. Heat currents are therefore strongly suppressed.

entropy current flow. The electron trapped by multiple Andreev reflections between the two superconductors continues its Hamiltonian time evolution in a definite quantum state, never entering the reservoir, and therefore generates no entropy. Substituting superconducting for normal metal contacts makes clear that entropy generation in the normal point contact actually occurs when the electron enters the measurement reservoirs.

8 Conclusions

Calcaulating the entropy current flow in a mesoscopic conductor is essential to understand its thermoelectric properties, such as the discretized thermopower of a quantum point contact. Placing a scatterer inside the point contact generates additional entropy and heat currents. This heat current flow raises fundamental physical questions about entropy and information flows in an open system. We showed that the mutual information flow between source and drain in the point contact is always larger than the physical entropy flow when a scattering obstacle is present. Replacing the normal metal contacts with superconducting ones eliminates the entropy current flow, showing that the entropy production in a mesoscopic conductor actually occurs in the measurement reservoirs attached to the conductor.

Acknowledgements

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