Demonstration of GALG tool

Presented for ANPA 2020

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Installing GALG Tool

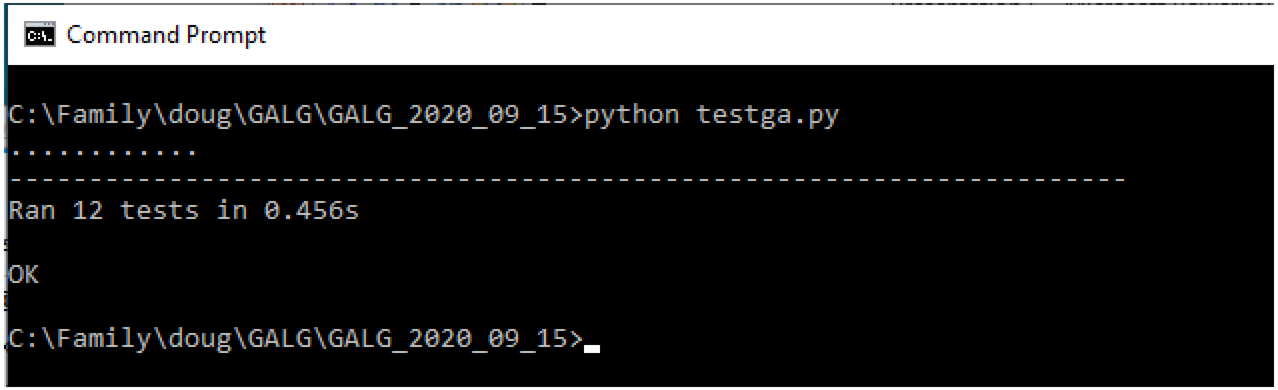
Download and install the 64bit version of python 2.7.18:

https://www.python.org/downloads/release/python-2718/

Download and unzip my GALG tool from this link:

http://www.matzkefamily.net/doug/GALG/GALG_2020_09_15.zip

Run the self test using command prompt tool:



Vectors & Multivectors using REP loop

Command Prompt - python -i ga.py

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i ga.py
>>> a
+ a
>>> b
+ b
>>> c
+ c
>>> a^b
+ (a^b)
>>> b^a
- (a^b)
>>> (1+a)(1+b)
+ 1 + a + b + (a^b)
>>> (1+a)(1+b)(1+c)(1+d)
+ 1 + a + b + c + d + (a^b) + (a^c) + (a^d) + (b^c) + (b^d) + (c^d) + (a^b^c) + (a^b^d) + (a^c^d) + (b^c^d) + (a^b^c^d)
>>> gastates((1+a)(1+b)(1+c)(1+d))
<table for + 1 + a + b + c + d + (a^b) + (a^c) + (a^d) + (b^c) + (b^d) + (c^d) + (a^b^c) + (a^b^d) + (a^c^d) + (b^c^d) + (a^b^c^d)>
INPUTS: a b c d | OUTPUT
------------------------
ROW 00: - - - - | 0
ROW 01: - - - + | 0
ROW 02: - - + - | 0
ROW 03: - - + + | 0
------------------------
ROW 04: - + - - | 0
ROW 05: - + - + | 0
ROW 06: - + + - | 0
ROW 07: - + + + | 0
------------------------
ROW 08: + - - - | 0
ROW 09: + - - + | 0
ROW 10: + - + - | 0
ROW 11: + - + + | 0
------------------------
ROW 12: + + - - | 0
ROW 13: + + - + | 0
ROW 14: + + + - | 0
ROW 15: + + + + | +
------------------------
Counts for outputs of ZERO=15, PLUS=1, MINUS=0 for TOTAL=16 rows
>>> _
```

Rock, Paper, Scissor, Lizard, Spock

Command Prompt

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i rock_paper_scissors.py
+ (p1p^p2r) - (p1p^p2s) - (p1r^p2p) + (p1r^p2s) + (p1s^p2p) - (p1s^p2r)
play_rps(r/p/s, r/p/s)
>>> play_rps('r', 'r')
'Tie'
>>> play_rps('r', 's')
'Player1'
>>> play_rps('r', 'p')
'Player2'
>>> ^Z


C:\Family\doug\GALG\GALG_2020_09_15>python -i rock_paper_scissors_lizard_spock.py
- (p1k^p2p) + (p1k^p2r) + (p1k^p2s) - (p1k^p2z) + (p1p^p2k) + (p1p^p2r) - (p1p^p2s) - (p1p^p2z) - (p1r^p2k)
1s^p2k) + (p1s^p2p) - (p1s^p2r) + (p1s^p2z) + (p1z^p2k) + (p1z^p2p) - (p1z^p2r) - (p1z^p2s)
play_rpszk(r/p/s/z/k, r/p/s/z/k)
>>> play_rpszk('r', 'p')
'Player2'
>>> play_rpszk('z', 'z')
'Tie'
>>> play_rpszk('z', 'r')
'Player2'
>>> play_rpszk('z', 'p')
'Player1'
>>> play_rpszk('z', 's')
'Player2'
>>> ^Z
```

Qubits using GALG

Command Prompt - python -i qubits.py

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i qubits.py
>>> A
+ a0 - a1
>>> B
+ b0 - b1
>>> A*B
+ (a0^b0) - (a0^b1) - (a1^b0) + (a1^b1)
>>> gastates(A)
<table for + a0 - a1>
INPUTS: a0 a1 | OUTPUT
----------------------
ROW 00: - - | 0
ROW 01: - + | +
ROW 02: + - | -
ROW 03: + + | 0
----------------------
Counts for outputs of ZERO=2, PLUS=1, MINUS=1 for TOTAL=4 rows
>>> gastates(A*B)
<table for + (a0^b0) - (a0^b1) - (a1^b0) + (a1^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 00: - - - - | 0
ROW 01: - - - + | 0
ROW 02: - - + - | 0
ROW 03: - - + + | 0
----------------------------
ROW 04: - + - - | 0
ROW 05: - + - + | +
ROW 06: - + + - | -
ROW 07: - + + + | 0
----------------------------
ROW 08: + - - - | 0
ROW 09: + - - + | -
ROW 10: + - + - | +
ROW 11: + - + + | 0
----------------------------
ROW 12: + + - - | 0
ROW 13: + + - + | 0
ROW 14: + + + - | 0
ROW 15: + + + + | 0
----------------------------
Counts for outputs of ZERO=12, PLUS=2, MINUS=2 for TOTAL=16 rows
>>>
```

Spinors and Qubits

Command Prompt - python -i qubits.py

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i qubits.py
>>> gastates(Sa)
<table for + (a0^a1)>
INPUTS: a0 a1 | OUTPUT
----------------------
ROW 00: - - | +
ROW 01: - + | -
ROW 02: + - | -
ROW 03: + + | +
----------------------
Counts for outputs of ZERO=0, PLUS=2, MINUS=2 for TOTAL=4 rows
>>> Sa**2
-1
>>> Sa**4
1
>>> gastates(A*Sa)
<table for + a0 + a1>
INPUTS: a0 a1 | OUTPUT
----------------------
ROW 00: - - | +
ROW 01: - + | 0
ROW 02: + - | 0
ROW 03: + + | -
----------------------
Counts for outputs of ZERO=2, PLUS=1, MINUS=1 for TOTAL=4 rows
>>> gastates(A*Sa*Sa)
<table for - a0 + a1>
INPUTS: a0 a1 | OUTPUT
----------------------
ROW 00: - - | 0
ROW 01: - + | -
ROW 02: + - | +
ROW 03: + + | 0
----------------------
Counts for outputs of ZERO=2, PLUS=1, MINUS=1 for TOTAL=4 rows
>>>
```

Roots of Unity

$X^2 = -1$ (*i*) and $X^2 = +1$ (neutrino)

```
>>> gasolve([a0, a1], lambda x: x**2, -1)
Found match at 12 where X=+ a0 + a1 produces -1 for both
Found match at 15 where X=- a0 + a1 produces -1 for both
Found match at 21 where X=+ a0 - a1 produces -1 for both
Found match at 24 where X=- a0 - a1 produces -1 for both
Found match at 27 where X=+ (a0^a1) produces -1 for both
Found match at 54 where X=- (a0^a1) produces -1 for both
Attempted 81 with 6 found.
<gasolve for [+ a0, + a1] tried=81 found=6>
>>> gasolve([a0, a1], lambda x: x**2, 1)
Found match at 1 where X=1 produces 1 for both
Found match at 2 where X=-1 produces 1 for both
Found match at 3 where X=+ a0 produces 1 for both
Found match at 6 where X=- a0 produces 1 for both
Found match at 9 where X=+ a1 produces 1 for both
Found match at 18 where X=- a1 produces 1 for both
Found match at 39 where X=+ a0 + a1 + (a0^a1) produces 1
Found match at 42 where X=- a0 + a1 + (a0^a1) produces 1
Found match at 48 where X=+ a0 - a1 + (a0^a1) produces 1
Found match at 51 where X=- a0 - a1 + (a0^a1) produces 1
Found match at 66 where X=+ a0 + a1 - (a0^a1) produces 1
Found match at 69 where X=- a0 + a1 - (a0^a1) produces 1
Found match at 75 where X=+ a0 - a1 - (a0^a1) produces 1
Found match at 78 where X=- a0 - a1 - (a0^a1) produces 1
Attempted 81 with 14 found.
<gasolve for [+ a0, + a1] tried=81 found=14>
```

$X^3 = +1$ (trine) and $X^4 = +1$ (sqrt not)

```
>>> gasolve([a0, a1], lambda x: x**3, 1)
Found match at 1 where X=1 produces 1 for both
Found match at 31 where X=+ 1 + a0 + (a0^a1) produces 1 for both
Found match at 34 where X=+ 1 - a0 + (a0^a1) produces 1 for both
Found match at 37 where X=+ 1 + a1 + (a0^a1) produces 1 for both
Found match at 46 where X=+ 1 - a1 + (a0^a1) produces 1 for both
Found match at 58 where X=+ 1 + a0 - (a0^a1) produces 1 for both
Found match at 61 where X=+ 1 - a0 - (a0^a1) produces 1 for both
Found match at 64 where X=+ 1 + a1 - (a0^a1) produces 1 for both
Found match at 73 where X=+ 1 - a1 - (a0^a1) produces 1 for both
Attempted 81 with 9 found.
<gasolve for [+ a0, + a1] tried=81 found=9>
>>> gasolve([a0, a1], lambda x: x**4, 1)
Found match at 1 where X=1 produces 1 for both
Found match at 2 where X=-1 produces 1 for both
Found match at 3 where X=+ a0 produces 1 for both
Found match at 6 where X=- a0 produces 1 for both
Found match at 9 where X=+ a1 produces 1 for both
Found match at 12 where X=+ a0 + a1 produces 1 for both
Found match at 15 where X=- a0 + a1 produces 1 for both
Found match at 18 where X=- a1 produces 1 for both
Found match at 21 where X=+ a0 - a1 produces 1 for both
Found match at 24 where X=- a0 - a1 produces 1 for both
Found match at 27 where X=+ (a0^a1) produces 1 for both
Found match at 39 where X=+ a0 + a1 + (a0^a1) produces 1 for both
Found match at 42 where X=- a0 + a1 + (a0^a1) produces 1 for both
Found match at 48 where X=+ a0 - a1 + (a0^a1) produces 1 for both
Found match at 51 where X=- a0 - a1 + (a0^a1) produces 1 for both
Found match at 54 where X=- (a0^a1) produces 1 for both
Found match at 66 where X=+ a0 + a1 - (a0^a1) produces 1 for both
Found match at 69 where X=- a0 + a1 - (a0^a1) produces 1 for both
Found match at 75 where X=+ a0 - a1 - (a0^a1) produces 1 for both
Found match at 78 where X=- a0 - a1 - (a0^a1) produces 1 for both
Attempted 81 with 20 found.
<gasolve for [+ a0, + a1] tried=81 found=20>
```

$\mathbf{X^2 = -X}$

```
>>> gasolve([a0, a1], lambda x: x**2, lambda x: -x)
Found match at 0 where X=0 produces 0 for both
Found match at 2 where X=-1 produces 1 for both
Found match at 4 where X=+ 1 + a0 produces - 1 - a0 for both
Found match at 7 where X=+ 1 - a0 produces - 1 + a0 for both
Found match at 10 where X=+ 1 + a1 produces - 1 - a1 for both
Found match at 19 where X=+ 1 - a1 produces - 1 + a1 for both
Found match at 40 where X=+ 1 + a0 + a1 + (a0^a1) produces - 1 - a0 - a1 - (a0^a1)
```

Nilpotents, Unitary and Idempotents

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i qubits.py
>>> gasolve([a0, a1], lambda x: x**2, 0)
Found match at 0 where X=0 produces 0 for both
Found match at 30 where X=+ a0 + (a0^a1) produces 0 for both
Found match at 33 where X=- a0 + (a0^a1) produces 0 for both
Found match at 36 where X=+ a1 + (a0^a1) produces 0 for both
Found match at 45 where X=- a1 + (a0^a1) produces 0 for both
Found match at 57 where X=+ a0 - (a0^a1) produces 0 for both
Found match at 60 where X=- a0 - (a0^a1) produces 0 for both
Found match at 63 where X=+ a1 - (a0^a1) produces 0 for both
Found match at 72 where X=- a1 - (a0^a1) produces 0 for both
Attempted 81 with 9 found.
<gasolve for [+ a0, + a1] tried=81 found=9>
>>> gasolve([a0, a1], lambda x: x**2, 1)
Found match at 1 where X=1 produces 1 for both
Found match at 2 where X=-1 produces 1 for both
Found match at 3 where X=+ a0 produces 1 for both
Found match at 6 where X=- a0 produces 1 for both
Found match at 9 where X=+ a1 produces 1 for both
Found match at 18 where X=- a1 produces 1 for both
Found match at 39 where X=+ a0 + a1 + (a0^a1) produces 1 for both
Found match at 42 where X=- a0 + a1 + (a0^a1) produces 1 for both
Found match at 48 where X=+ a0 - a1 + (a0^a1) produces 1 for both
Found match at 51 where X=- a0 - a1 + (a0^a1) produces 1 for both
Found match at 66 where X=+ a0 + a1 - (a0^a1) produces 1 for both
Found match at 69 where X=- a0 + a1 - (a0^a1) produces 1 for both
Found match at 75 where X=+ a0 - a1 - (a0^a1) produces 1 for both
Found match at 78 where X=- a0 - a1 - (a0^a1) produces 1 for both
Attempted 81 with 14 found.
<gasolve for [+ a0, + a1] tried=81 found=14>
>>> gasolve([a0, a1], lambda x: x**2, lambda x: x)
Found match at 0 where X=0 produces 0 for both
Found match at 1 where X=1 produces 1 for both
Found match at 5 where X=- 1 + a0 produces - 1 + a0 for both
Found match at 8 where X=- 1 - a0 produces - 1 - a0 for both
Found match at 11 where X=- 1 + a1 produces - 1 + a1 for both
Found match at 20 where X=- 1 - a1 produces - 1 - a1 for both
Found match at 41 where X=- 1 + a0 + a1 + (a0^a1) produces - 1 + a0 + a1 + (a0^a1) for both
Found match at 44 where X=- 1 - a0 + a1 + (a0^a1) produces - 1 - a0 + a1 + (a0^a1) for both
Found match at 50 where X=- 1 + a0 - a1 + (a0^a1) produces - 1 + a0 - a1 + (a0^a1) for both
Found match at 53 where X=- 1 - a0 - a1 + (a0^a1) produces - 1 - a0 - a1 + (a0^a1) for both
Found match at 68 where X=- 1 + a0 + a1 - (a0^a1) produces - 1 + a0 + a1 - (a0^a1) for both
Found match at 71 where X=- 1 - a0 + a1 - (a0^a1) produces - 1 - a0 + a1 - (a0^a1) for both
Found match at 77 where X=- 1 + a0 - a1 - (a0^a1) produces - 1 + a0 - a1 - (a0^a1) for both
Found match at 80 where X=- 1 - a0 - a1 - (a0^a1) produces - 1 - a0 - a1 - (a0^a1) for both
Attempted 81 with 14 found.
<gasolve for [+ a0, + a1] tried=81 found=14>
```

For $\mathbf{X}^2 = \mathbf{X}$ (Idempotent) and $\mathbf{U}^2 = 1$ (Unitary)

then $\mathbf{X} = -1 \pm \mathbf{U}$

proof

$\mathbf{X}^2 = (-1 \pm \mathbf{U})^2 = \mathbf{X}$)

Qubit Summary G_2

Another 40 are additive inverses of these 40 and have the same properties

Table 7.2: Operator Summary for 41 out of 81 states for Q_1

	Equation combinations X by Cartesian Distance	Cart Dist	Eqn Label	$-X$	$(X)^{-1}$	X^2	$\sqrt{X}$	Comp Basis Vect
bit-vector spinor	0	0	0000	0000	none	0	Found 8	[0 0 0 0]
	- 1	1	000-	000+	x	+1	Found 6	[- - - -]
	- a0	1	00-0	00+0	x	+1	none	[+ + - -]
	- a1	1	0-00	0+00	x	+1	none	[+ - + -]
	- a0 a1	1	-000	+000	-x	-1	±00±	[- + + -]
idempotent	- 1 - a0 = I^+	Cart Dist from 0 is $\sqrt{2}$	00--	00++	none	+x	±x	[0 0 + +]
	+ 1 - a0 = I^-		00-+	00+-	none	-x	none	[- - 0 0]
	- 1 - a1 = I^+		0-0-	0+0+	none	+x	±x	[0 + 0 +]
	+ 1 - a1 = I^-		0-0+	0+0-	none	-x	none	[- 0 - 0]
qubit	- a0 - a1		0--0	0++0	-x	-1	0∓±±	[- 0 0 +]
	+ a0 - a1		0-+0	0+-0	-x	-1	0±±±	[0 + - 0]
Pauli spin	- 1 - a0 a1 = I^+		-00-	+00+	-00+	-000	none	[+ 0 0 +]
	+ 1 - a0 a1 = I^-		-00+	+00-	-00-	+000	none	[0 - - 0]
boson	- a0 - a0 a1		-0-0	+0+0	none	0	none	[0 - 0 +]
	+ a0 - a0 a1		-0+0	+0-0	none	0	none	[+ 0 - 0]
	- a1 - a0 a1		--00	++00	none	0	none	[0 0 - +]
	+ a1 - a0 a1		-+00	+-00	none	0	none	[+ - 0 0]
qubit ±1	- 1 - a0 - a1	Cart Dist from 0 is $\sqrt{3}$	0---	0+++	0--+	0--0	none	[+ - - 0]
	+ 1 - a0 - a1		0--+	0++-	0---	0++0	none	[0 + + -]
	- 1 + a0 - a1		0-+-	0+-+	0-++	0-+0	none	[- 0 + -]
	+ 1 + a0 - a1		0-++	0+--	0-+-	0+-0	none	[+ - 0 +]
neutrino/antineutrino	- a0 - a1 - a0 a1		---0	+++0	x	+1	none	[+ + + 0]
	+ a0 - a1 - a0 a1		--+0	++-0	x	+1	none	[- - 0 -]
	- a0 + a1 - a0 a1		-+-0	+-+0	x	+1	none	[- 0 - -]
	+ a0 + a1 - a0 a1		-++0	+--0	x	+1	none	[0 + + +]
trine and boson -1	- 1 - a0 - a0 a1		-0--	+0++	+0+-	-0-+	none	[- + - 0]
	+ 1 - a0 - a0 a1		-0-+	+0+-	+0++	+0++	±0± ±	[+ 0 + -]
	- 1 + a0 - a0 a1		-0+-	+0-+	+0--	-0++	none	[0 - + -]
	+ 1 + a0 - a0 a1		-0++	+0--	+0-+	+0-+	±0∓±	[- + 0 +]
	- 1 - a1 - a0 a1		--0-	++0+	++0-	--0+	none	[- - + 0]
	+ 1 - a1 - a0 a1		--0+	++0-	++0+	++0+	±±0±	[+ + 0 -]
	- 1 + a1 - a0 a1		-+0-	+-0+	+-0-	-+0+	none	[0 + - -]
	+ 1 + a1 - a0 a1		-+0+	+-0-	+-0+	+-0+	±∓0±	[- 0 + +]
neutrino ±1	- 1 - a0 - a1 - a0 a1	Cart Dist from 0 is $\sqrt{4}$	----	++++	none	+x	±x	[0 0 0 -]
	+ 1 - a0 - a1 - a0 a1		---+	+++-	none	-x	none	[- - - +]
	- 1 + a0 - a1 - a0 a1		--+-	++-+	none	+x	±x	[+ + - +]
	+ 1 + a0 - a1 - a0 a1		--++	++--	none	-x	none	[0 0 + 0]
	- 1 - a0 + a1 - a0 a1		-+--	+-++	none	+x	±x	[+ - + +]
	+ 1 - a0 + a1 - a0 a1		-+-+	+-+-	none	-x	none	[0 + 0 0]
	- 1 + a0 + a1 - a0 a1		-++-	+--+	none	+x	±x	[- 0 0 0]
	+ 1 + a0 + a1 - a0 a1		-+++	+---	none	-x	none	[+ - - -]

Four Neutrinos in G_2

```
****************************here are 8 neutrinos for G2
>>> for q in make_neutrinos(): report2(q)
...
1.755 ((0, 1, 3), 3) [0 - - -] = - a - b + (a^b)
1.755 ((0, 1, 3), 3) [- 0 - -] = - a + b - (a^b)
1.755 ((0, 1, 3), 3) [- - 0 -] = + a - b - (a^b)
1.755 ((0, 1, 3), 3) [- - - 0] = + a + b + (a^b)
1.755 ((0, 1, 3), 3) [+ + + 0] = - a - b - (a^b)
1.755 ((0, 1, 3), 3) [+ 0 + +] = + a - b + (a^b)
1.755 ((0, 1, 3), 3) [+ + 0 +] = - a + b + (a^b)
1.755 ((0, 1, 3), 3) [0 + + +] = + a + b - (a^b)
```

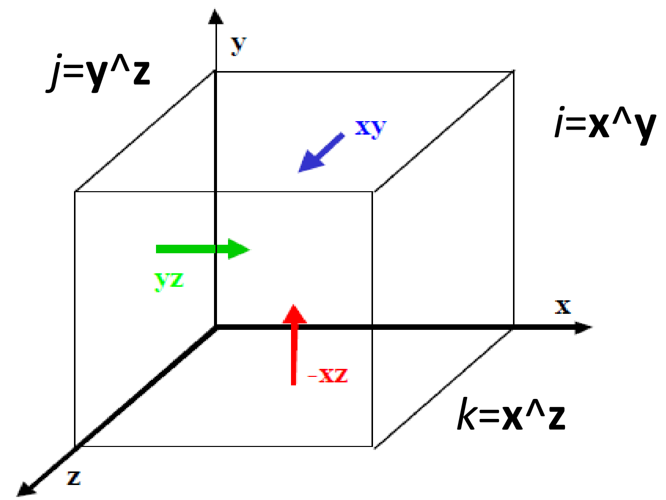
Name	*Form*	*Vector* ($\mathcal{G}_2$)	*Signature*	*Bits*
ν	$a+b+ab$	$[---0]$	$(0,1,3),3$	1.75
ν_μ	$a-b-ab$	$[--0-]$	"	"
ν_τ	$-a+b-ab$	$[-0--]$	"	"
$\Sigma =$	$a+b-ab$	$[0+++]$	"	"
$\bar{\nu}$	$-a-b-ab$	$[+++0]$	"	"
$\bar{\nu}_\mu$	$-a+b+ab$	$[++0+]$	"	"
$\bar{\nu}_\tau$	$a-b+ab$	$[+0++]$	"	"
$\Sigma =$	$-a-b+ab$	$[0---]$	"	"

Table 4.13: Eigenvector Summary from $E_k R_k = R_k$ for $\mathcal{G}_2$

Primary Basis Set				Dual Basis Set			
k =	$E_k = R_k - 1$	$P_k = -R_k$	$R_k = 1 + E_k$	k =	$E_k = R_k - 1$	$P_k = -R_k$	$R_k = 1 + E_k$
0	$[0---]$	$[-000]$	$[+000]$	7	$[0+++]$	$[-+++]$	$[+---]$
1	$[-0--]$	$[0-00]$	$[0+00]$	6	$[+0++]$	$[+-++]$	$[-+--]$
2	$[--0-]$	$[00-0]$	$[00+0]$	5	$[++0+]$	$[++-+]$	$[--+-]$
3	$[---0]$	$[000-]$	$[000+]$	4	$[+++0]$	$[+++-]$	$[---+]$
sum	$[0000]$	$[----]$	$[++++]$	sum	$[0000]$	$[----]$	$[++++]$

Quaternions in G_3

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i ga.py
>>> Qi=x^y
>>> Qj=y^z
>>> Qk=x^z
>>> (Qi+Qj+Qk)
+ (x^y) + (x^z) + (y^z)
>>> (Qi+Qj+Qk)**2
0
>>> (Qi*Qj*Qk)
-1
>>> Qi*Qj, Qk
(+ (x^z), + (x^z))
>>> Qj*Qk, Qi
(+ (x^y), + (x^y))
>>> Qk*Qi, Qj
(+ (y^z), + (y^z))
>>> Qk**2
-1
```



quaternions {*i*, *j*, *k*}

The quaternion sum is nilpotent!

(Qi+Qj+Qk)**2 = 0

Bell Operator: concurrent spinors

Command Prompt - python -i qubits.py

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i qubits.py
>>> gastates(Sa)
<table for + (a0^a1)>
INPUTS: a0 a1 | OUTPUT
----------------------
ROW 00: - - | +
ROW 01: - + | -
ROW 02: + - | -
ROW 03: + + | +
----------------------
Counts for outputs of ZERO=0, PLUS=2, MINUS=2 for TOTAL=4 rows
>>> gastates(Sa+Sb, zeros=False)
<table for + (a0^a1) + (b0^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 00: - - - - | -
ROW 03: - - + + | -
----------------------------
ROW 05: - + - + | +
ROW 06: - + + - | +
----------------------------
ROW 09: + - - + | +
ROW 10: + - + - | +
----------------------------
ROW 12: + + - - | -
ROW 15: + + + + | -
----------------------------
Counts for outputs of ZERO=8, PLUS=4, MINUS=4 for TOTAL=16 rows
>>>
```

Entanglement: Multiplicative Cancellation

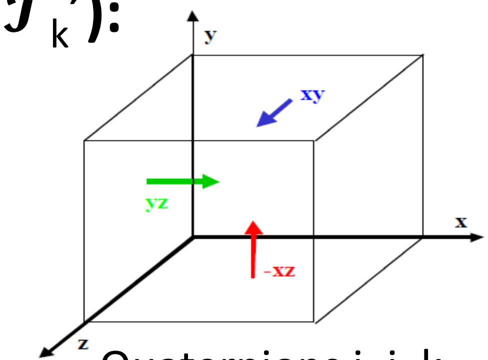
Command Prompt - python -i qubits.py

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i qubits.py
>>> gastates(A*B, zeros=False)
<table for + (a0^b0) - (a0^b1) - (a1^b0) + (a1^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 05: - + - + | +
ROW 06: - + + - | -
----------------------------
ROW 09: + - - + | -
ROW 10: + - + - | +
----------------------------
Counts for outputs of ZERO=12, PLUS=2, MINUS=2 for TOTAL=16 rows
>>> gastates(A*B*(Sa+Sb), zeros=False)
<table for - (a0^b0) + (a1^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 01: - - - + | +
ROW 02: - - + - | -
----------------------------
ROW 04: - + - - | +
ROW 07: - + + + | -
----------------------------
ROW 08: + - - - | -
ROW 11: + - + + | +
----------------------------
ROW 13: + + - + | -
ROW 14: + + + - | +
----------------------------
Counts for outputs of ZERO=8, PLUS=4, MINUS=4 for TOTAL=16 rows
>>>
```

TauQuernions: Entangled Quaternions in $\mathbb{G}_4$

➢ **TauQuernions ($\mathcal{T}_i$, $\mathcal{T}_j$, $\mathcal{T}_k$ & conjugate set $\mathcal{T}_i'$, $\mathcal{T}_j'$, $\mathcal{T}_k'$):**

- Entangled Quaternion isomorphs
- $\mathcal{M} = \mathcal{T}_i$ = **ab – cd**, $\mathcal{T}_j$ = **ac + bd** and $\mathcal{T}_k$ = **ad – bc**
- $\mathcal{B} = \mathcal{T}_i'$= **ab + cd**, $\mathcal{T}_j'$= **ac – bd** and $\mathcal{T}_k'$= **ad + bc**
- Anti-Commutative: $\mathcal{T}_x \mathcal{T}_y = -\mathcal{T}_x \mathcal{T}_y$
- $\mathcal{T}_i^2 = \mathcal{T}_j^2 = \mathcal{T}_k^2 = \mathcal{T}_i \mathcal{T}_j \mathcal{T}_k = I^- = (1 + \mathbf{abcd})$ (sparse –1)
- $(I^-)^2 = I^+ = (-1 \pm \mathbf{abcd})$ (sparse +1: is idempotent)



Quaternions i, j, k: {xy, yz, xz}

```
>>> report4(1-abcd)
18.868 ((0, 8, 8), 1) [0 - - 0 - 0 0 - - 0 0 - 0 - - 0] = + 1 - (a^b^c^d)
>>> report4(-1-abcd)
18.868 ((0, 8, 8), 1) [+ 0 0 + 0 + + 0 0 + + 0 + 0 0 +] = - 1 - (a^b^c^d)
```

$\mathcal{B}^2 + \mathcal{M}^2 = -1$

$\mathcal{B}^4 + \mathcal{M}^4 = +1$

*	$\mathcal{T}_i$	$\mathcal{T}_j$	$\mathcal{T}_k$
$\mathcal{T}_i$	1 + **abcd**	–**ad** + **bc**	**ac** + **bd**
$\mathcal{T}_j$	**ad** – **bc**	1 + **abcd**	–**ab** + **cd**
$\mathcal{T}_k$	–**ac** – **bd**	**ab** – **cd**	1 + **abcd**

*	$\mathcal{T}_i$	$\mathcal{T}_y$	$\mathcal{T}_k$
$\mathcal{T}_i$	"–1"	$-\mathcal{T}_k$	$\mathcal{T}_j$
$\mathcal{T}_j$	$\mathcal{T}_k$	"–1"	$-\mathcal{T}_i$
$\mathcal{T}_k$	$-\mathcal{T}_j$	$\mathcal{T}_i$	"–1"

$\mathcal{T}_i$	$\mathcal{T}_j$	$\mathcal{T}_k$
Magic	$M_3 = -M_1$	$M_0 = -M_2$
Magic	$M_3 = -M_1$	$M_2 = -M_0$
Magic	$M_1 = -M_3$	$M_0 = -M_2$
Magic	$M_1 = -M_3$	$M_2 = -M_0$

$\mathcal{T}_i'$	$\mathcal{T}_j'$	$\mathcal{T}_k'$
Bell	$B_2 = -B_0$	$B_1 = -B_3$
Bell	$B_2 = -B_0$	$B_3 = -B_1$
Bell	$B_0 = -B_2$	$B_1 = -B_3$
Bell	$B_0 = -B_2$	$B_3 = -B_1$

$\mathcal{B}$ and $\mathcal{M}$ operators are used as states

Tauquernions

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i tau_code.py
***************************
Tx= + (a^b) - (c^d) [0 - - 0 + 0 0 + + 0 0 + 0 - - 0]
Ty= + (a^c) + (b^d) [- 0 0 + 0 - + 0 0 + - 0 + 0 0 -]
Tz= + (a^d) - (b^c) [0 + - 0 - 0 0 + + 0 0 - 0 - + 0]
***************************
Tx**2 = Sparse -1 =  + 1 + (a^b^c^d) [- 0 0 - 0 - - 0 0 - - 0 - 0 0 -]
Ty**2 = Sparse -1 =  + 1 + (a^b^c^d) [- 0 0 - 0 - - 0 0 - - 0 - 0 0 -]
Tz**2 = Sparse -1 =  + 1 + (a^b^c^d) [- 0 0 - 0 - - 0 0 - - 0 - 0 0 -]
***************************
-1 -abcd = Sparse +1 = - Magic**2 = Tx*Ty*Tz =  - 1 - (a^b^c^d) [+ 0 0 + 0 + + 0 0 + + 0 + 0 0 +]
-1 +abcd = Sparse +1 = - Bell**2  - 1 + (a^b^c^d) [0 + + 0 + 0 0 + + 0 0 + 0 + + 0]
+1 +abcd = Sparse -1 = Magic**2 =  + 1 + (a^b^c^d) [- 0 0 - 0 - - 0 0 - - 0 - 0 0 -]
+1 -abcd = Sparse -1 = Bell**2 =  + 1 - (a^b^c^d) [0 - - 0 - 0 0 - - 0 0 - 0 - - 0]
*************************** these are higgs
+ (a^b) + (a^c) + (a^d) + (b^c) - (b^d) + (c^d) where H*H= 0
+ (a^b) + (a^c) + (a^d) - (b^c) + (b^d) - (c^d) where H*H= 0
+ (a^b) + (a^c) - (a^d) + (b^c) + (b^d) - (c^d) where H*H= 0
+ (a^b) + (a^c) - (a^d) - (b^c) - (b^d) + (c^d) where H*H= 0
+ (a^b) - (a^c) + (a^d) + (b^c) + (b^d) + (c^d) where H*H= 0
+ (a^b) - (a^c) + (a^d) - (b^c) - (b^d) - (c^d) where H*H= 0
+ (a^b) - (a^c) - (a^d) + (b^c) - (b^d) - (c^d) where H*H= 0
+ (a^b) - (a^c) - (a^d) - (b^c) + (b^d) + (c^d) where H*H= 0
- (a^b) + (a^c) + (a^d) + (b^c) - (b^d) - (c^d) where H*H= 0
- (a^b) + (a^c) + (a^d) - (b^c) + (b^d) + (c^d) where H*H= 0
- (a^b) + (a^c) - (a^d) + (b^c) + (b^d) + (c^d) where H*H= 0
- (a^b) + (a^c) - (a^d) - (b^c) - (b^d) - (c^d) where H*H= 0
- (a^b) - (a^c) + (a^d) + (b^c) + (b^d) - (c^d) where H*H= 0
- (a^b) - (a^c) + (a^d) - (b^c) - (b^d) + (c^d) where H*H= 0
- (a^b) - (a^c) - (a^d) + (b^c) - (b^d) + (c^d) where H*H= 0
- (a^b) - (a^c) - (a^d) - (b^c) + (b^d) - (c^d) where H*H= 0
```

Sparse Invariants for Bell/Magic operators

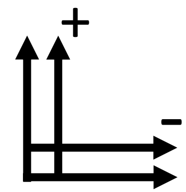
Bell Sparse Invariants

```
C:\Family\doug\GALG\GALG_2020_09_15>python -i qubits.py
>>> gastates((Sa+Sb)**2, zeros=False)
<table for + 1 - (a0^a1^b0^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 01: - - - + | -
ROW 02: - - + - | -
----------------------------
ROW 04: - + - - | -
ROW 07: - + + + | -
----------------------------
ROW 08: + - - - | -
ROW 11: + - + + | -
----------------------------
ROW 13: + + - + | -
ROW 14: + + + - | -
----------------------------
Counts for outputs of ZERO=8, PLUS=0, MINUS=8 for TOTAL=16 rows
>>> gastates((Sa+Sb)**4, zeros=False)
<table for - 1 + (a0^a1^b0^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 01: - - - + | +
ROW 02: - - + - | +
----------------------------
ROW 04: - + - - | +
ROW 07: - + + + | +
----------------------------
ROW 08: + - - - | +
ROW 11: + - + + | +
----------------------------
ROW 13: + + - + | +
ROW 14: + + + - | +
----------------------------
Counts for outputs of ZERO=8, PLUS=8, MINUS=0 for TOTAL=16 rows
```

Magic Sparse Invariants

```
>>> gastates((Sa-Sb)**2, zeros=False)
<table for + 1 + (a0^a1^b0^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 00: - - - - | -
ROW 03: - - + + | -
----------------------------
ROW 05: - + - + | -
ROW 06: - + + - | -
----------------------------
ROW 09: + - - + | -
ROW 10: + - + - | -
----------------------------
ROW 12: + + - - | -
ROW 15: + + + + | -
----------------------------
Counts for outputs of ZERO=8, PLUS=0,
>>> gastates((Sa-Sb)**4, zeros=False)
<table for - 1 - (a0^a1^b0^b1)>
INPUTS: a0 a1 b0 b1 | OUTPUT
----------------------------
ROW 00: - - - - | +
ROW 03: - - + + | +
----------------------------
ROW 05: - + - + | +
ROW 06: - + + - | +
----------------------------
ROW 09: + - - + | +
ROW 10: + - + - | +
----------------------------
ROW 12: + + - - | +
ROW 15: + + + + | +
----------------------------
Counts for outputs of ZERO=8, PLUS=8,
```

Ebits: Detailed Bell/Magic States

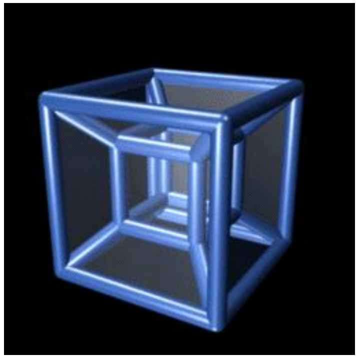


$\Phi^{\pm} = |00\rangle \pm |11\rangle$

$\Psi^{\pm} = |01\rangle \pm |10\rangle$

- Bell/Magic Operators (in $\mathbb{G}_4$):
  - **Bell** operator $\mathcal{B}$ = $\mathbf{S_A + S_B}$ = **a0^a1 + b0^b1**
  - **Magic** operator $\mathcal{M}$ = $\mathbf{S_A - S_B}$ = **a0^a1 – b0^b1**
- Bell/Magic operators $\mathcal{B}=\mathcal{B}^4$ and $\mathcal{M}=\mathcal{M}^4$ form ring states $\mathcal{B}_i$ and $\mathcal{M}_i$:

$B_{(i+1)mod4} = B_i\,(S_A + S_B)$	$M_{(i+1)mod4} = M_i\,(S_A - S_B)$
B_0 = $\mathbf{A_0\,B_0}$ **Bell** = $-\,\mathbf{S}_{00} + \mathbf{S}_{11} = \Phi^+$	M_0 = $\mathbf{A_0\,B_0}$ **Magic** = $+\,\mathbf{S}_{01} - \mathbf{S}_{10}$
$B_1 = B_0$ **Bell** = $+\,\mathbf{S}_{01} + \mathbf{S}_{10} = \Psi^+$	$M_1 = M_0$ **Magic** = $-\,\mathbf{S}_{00} - \mathbf{S}_{11}$
$B_2 = B_1$ **Bell** = $+\,\mathbf{S}_{00} - \mathbf{S}_{11} = \Phi^-$	$M_2 = M_1$ **Magic** = $-\,\mathbf{S}_{01} + \mathbf{S}_{10}$
$B_3 = B_2$ **Bell** = $-\,\mathbf{S}_{01} - \mathbf{S}_{10} = \Psi^-$	$M_3 = M_2$ **Magic** = $+\,\mathbf{S}_{00} + \mathbf{S}_{11}$
$B_0 = B_3$ **Bell** = $-\,\mathbf{S}_{00} + \mathbf{S}_{11} = \Phi^+$	$M_0 = M_3$ **Magic** = $+\,\mathbf{S}_{01} - \mathbf{S}_{10}$

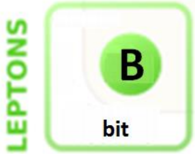


4D tesseract

- Cannot factor: ± **a0^b0** ± **a1^b1** (Inseparable and is singular)
- **Bell** and **Magic** operators are irreversible in $\mathbb{G}_4$ (different than Hilbert spaces)
  - See proofs that $1/(\mathbf{S_A} \pm \mathbf{S_B})$ does not exist for Bell (or Magic) operators
- Multiplicative Cancellation – *Information erasure is irreversible*
  - **Qubits** $\mathbf{A_0\,B_0}$ **= + a0^b0 – a0^b1 – a1^b0 + a1^b1** = $B_3 + M_3$
  - 0 = **Bell * Magic** = **Bell** * M_j = **Magic** * B_i = B_i * M_j
- Also works for higher dimensions $\mathcal{B}$ = $\mathbf{S_A} \pm \mathbf{S_B} \pm \mathbf{S_C} \pm \ldots$ (roots of unity)

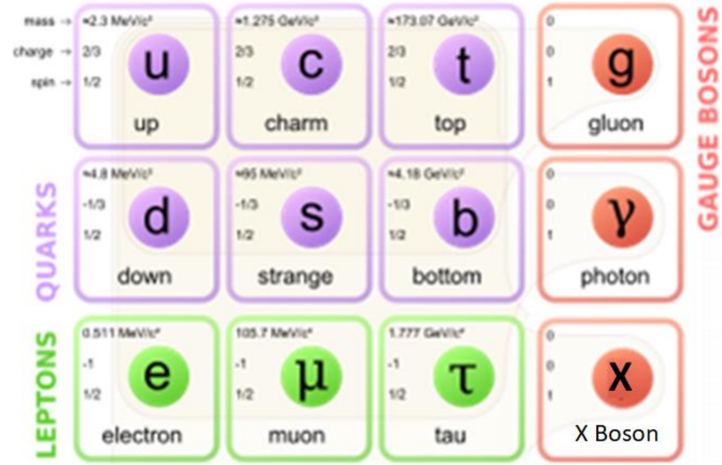
Graded Standard Model with GALG

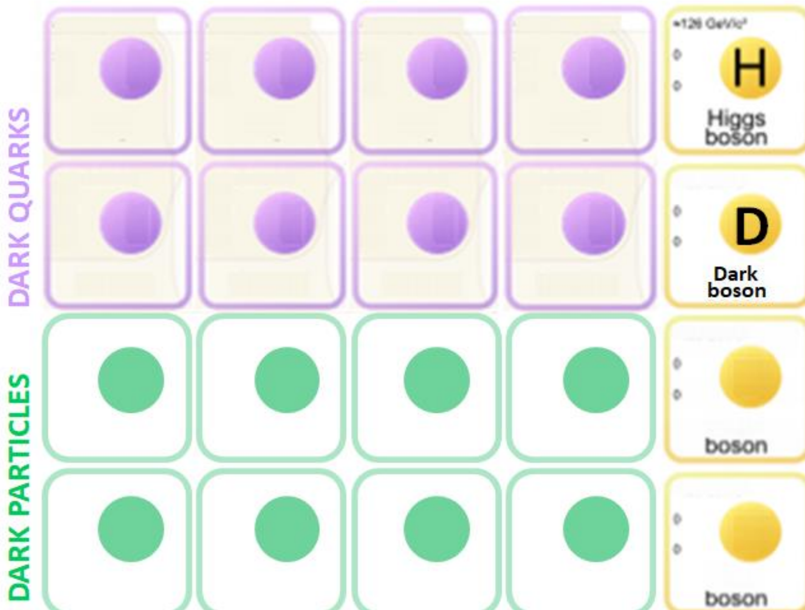
Bit in G_1



primitives in G_2 plus qubit

primitives in G_3 plus protons/neutrons





possible dark primitives in G_4

Question and Answers