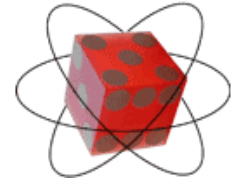




Business Issues Regarding Future Computers

Dallas Nanotechnology Focus Group
Nov 7, 2006

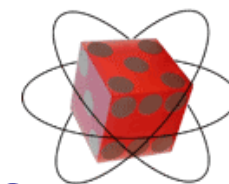
Douglas J. Matzke, Ph.D.
CTO of Syngence, LLC
Doug@QuantumDoug.com



Introduction and Outline

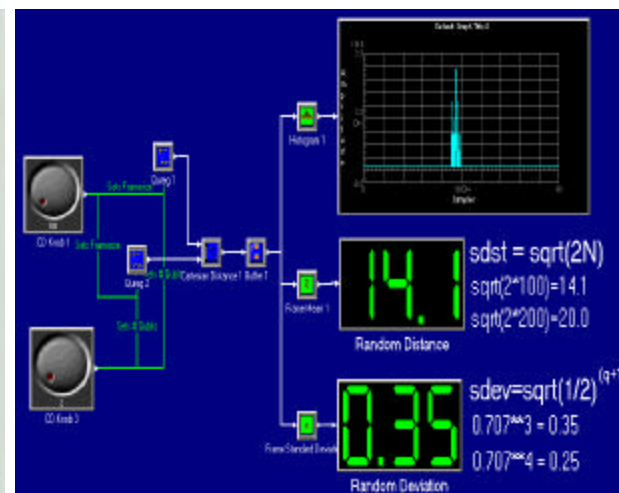
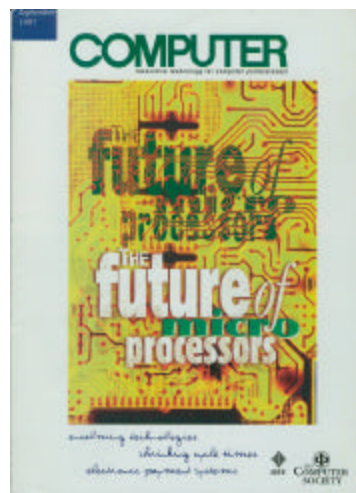
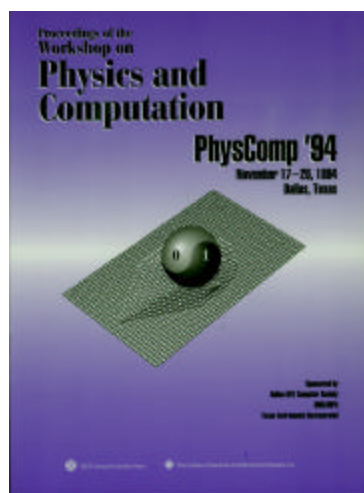
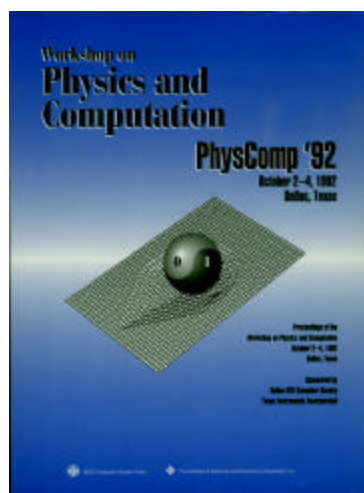
Topics in Presentation

- What does it take to build a GP computer?
- Limits of semiconductor/computer scaling
- Introduce idealized model of computational costs
- Introduce Quantum computing
- Information is Physical
- Compare/Contrast Classical Comp vs. QuComp
- Computing Myths
- Business Predictions
- Conclusions

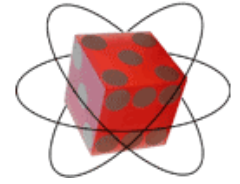


Motivation: Limits of Computation

- >25 Years in semiconductor company (HW/SW)
- PhysComp 1981, 1992, 1994, 1996 (chairman)
- Billion Transistor issue of Computer Sept 1997
- Ph.D in area of Quantum Computing May, 2002
- Quantum Computing Research contract 2003-2004



Conventional semiconductors will stop scaling in next 10+ years



End of Silicon Scaling

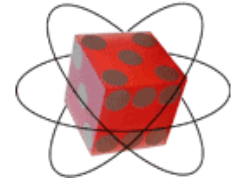
“Manufacturers will be able to produce chips on the 16-nanometer* manufacturing process, expected by conservative estimates to arrive in 2018, and maybe one or two manufacturing processes after that, but that's it.”

This is actually a power density/heat removal limit!!

Quote from News.com article “Intel scientists find wall for Moore’s Law” and Proc of IEEE Nov 2003 article: “Limits to Binary Logic Switch Scaling—A Gedanken Model”

*gate length of 9 nm, 93 W/cm² & 1.5x10² gates/cm²

ITRS: International Technology Roadmap for Semiconductors



Near-term Years

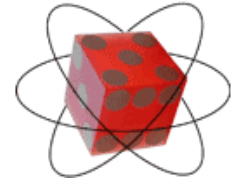
YEAR OF PRODUCTION	2003	2004	2005	2006	2007	2008	2009
Technology Node		hp90			hp65		
DRAM $\frac{1}{2}$ Pitch (nm)	100	90	80	70	65	57	50
MPU/ASIC M1 $\frac{1}{2}$ Pitch (nm)	120	107	95	85	75	67	60
MPU/ASIC Poly Si $\frac{1}{2}$ Pitch (nm)	107	90	80	70	65	57	50
MPU Printed Gate Length (nm)	65	53	45	40	35	32	28
MPU Physical Gate Length (nm)	45	37	32	28	25	22	20

Long-term Years

YEAR OF PRODUCTION	2010	2012	2013	2015	2016	2018
Technology Node	hp45		hp32		hp22	
DRAM $\frac{1}{2}$ Pitch (nm)	45	35	32	25	22	18
MPU/ASIC M1 $\frac{1}{2}$ Pitch (nm)	54	42	38	30	27	21
MPU/ASIC Poly Si $\frac{1}{2}$ Pitch (nm)	45	35	32	25	22	18
MPU Printed Gate Length (nm)	25	20	18	14	13	10
MPU Physical Gate Length (nm)	18	14	13	10	9	7

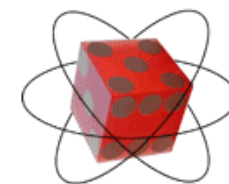
15 year forecast from 2003 ITRS - International Technology Roadmap for Semiconductors at: <http://www.itrs.net/>

These sizes are close to physical limits and technological limits.

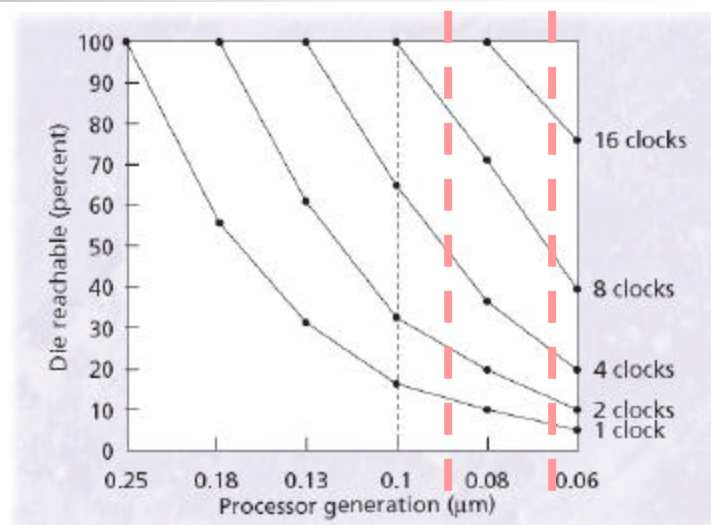
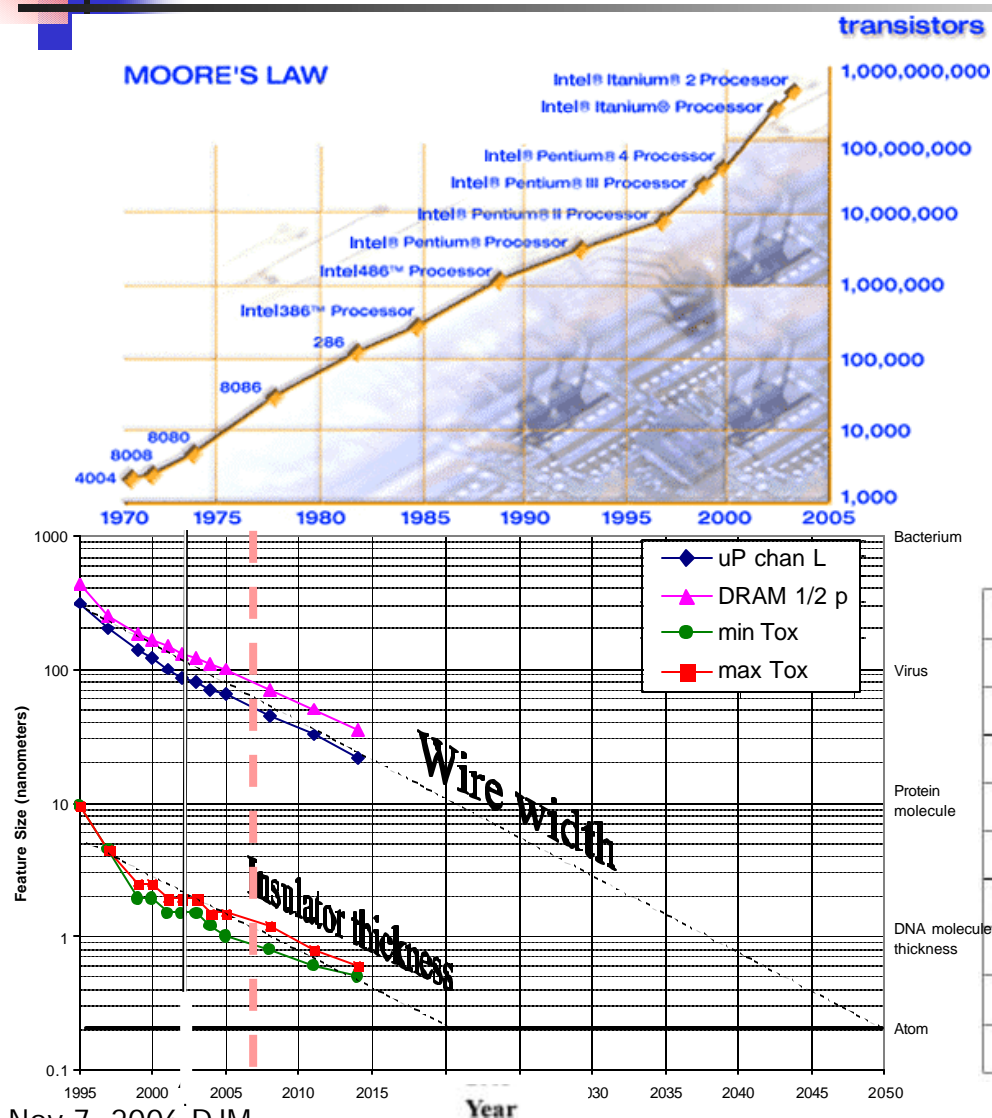


Computer Scaling Limits

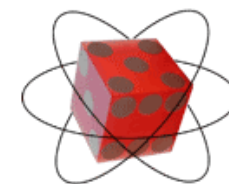
- Physical Limits
 - Power density/Dissipation: max is 100 W/cm²
 - Thermal/noise: $E/f = 100h$
 - Molecular/atomic/charge discreteness limits
 - Quantum: tunneling & Heisenberg uncertainty
- Technology Limits
 - Gate Length: min ~18-22 nm
 - Lithography Limits: wavelength of visible light
 - Power dissipation (100 watts) and Temperature
 - Wire Scaling: multicpu chips at ~ billion transistors
 - Materials



Charts and Tables Galore



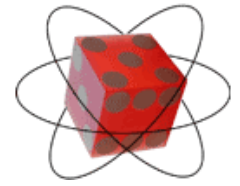
Year of Introduction	2001	2003	2006	2009
Minimum feature size (nm)	150	120	90	65
Overlay (nm)	55	45	35	25
Optical 193 nm	✓	✓		
Optical 157 nm		✓	✓	
Extreme Ultraviolet			✓	✓
X-rays				✓
Electron beam			✓	✓
Ion beam			✓	✓
Printing				✓



No Limits to Limits

- Space/Time/locality/Complexity limits
 - Architectures/circuits: logic/memory tradeoffs, Von Neumann
 - Algorithmic: sequential/parallel superscalar/vliw etc
 - Gate Fanin/Fanout and chip Pin/packaging limits
 - Communications Latency/bandwidth limits
 - Dimensionality Limits: pointers and interlinking
 - Clocking and Synchronization
 - Grain size: hw/sw/fpga
 - Noise/Error Correction
 - Deterministic vs. Probabilistic
 - Automatic Learning and meaning
 - Programming and representation: bits, qubits and ebits
 - NP Complete/hard: Black Hole threshold or age of universe.
 - ... etc
- Economic Limits
 - Research, fab build, wafer build, chip design, chip test, etc

What does it take to build a general purpose computer?



Computing is the time-evolution of physical systems.

- Model of Computation

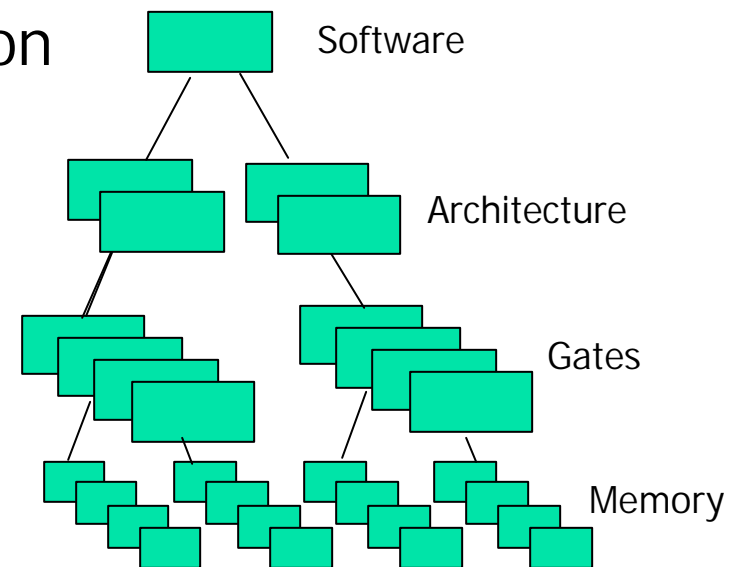
- Representation of Information
- Distinguishability of States
- Memory/Algorithms

- Physical Computers

- Matter/energy
- Space/time
- Noise/defect immunity

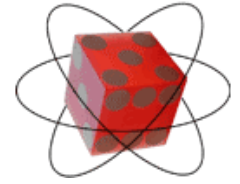
- Common Examples

- Classical Mechanical/Semiconductor
- Neurological/Biological/DNA
- Quantum Computer – a *Paradigm Shift*



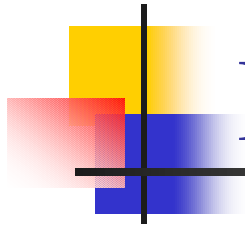


Introduce idealized model of computational costs

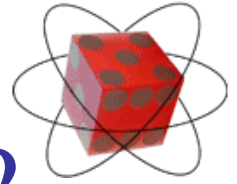


- *Space*: Information is in wrong place – Move it
 - Locality metrics are critical - context
 - Related to number of spatial dimensions - anisotropic
 - i.e. Busses, networks, caches, paging, regs, objects, ...
- *Time*: Information is in wrong form – Convert it
 - Change rate and parallelism are critical (locality)
 - Related to temporal reference frame (i.e. time dilation)
 - i.e. consistency, FFT, holograms, probabilities, wholism
- All other physical costs
 - Creation/Erasure, Noise/ECC, Uncertainty, Precision, ...
 - Decidability, Distinguishability, Detection, ...

See my paper on this subject from 1986

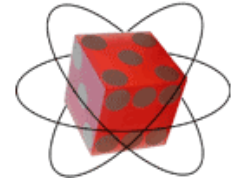


Idealized Smarter Computers?



- If Information is always in right “local” place(s)
 - Possible higher number of dimensions
 - Possible selective length contraction
- If Information is always in “correct” form(s)
 - Multiple consistent wholistic representations
 - Change occurs outside normal time
- If other costs mitigated
 - Arbitrarily high precision and distinguishability, etc
 - Arbitrarily low noise and uncertainty, etc

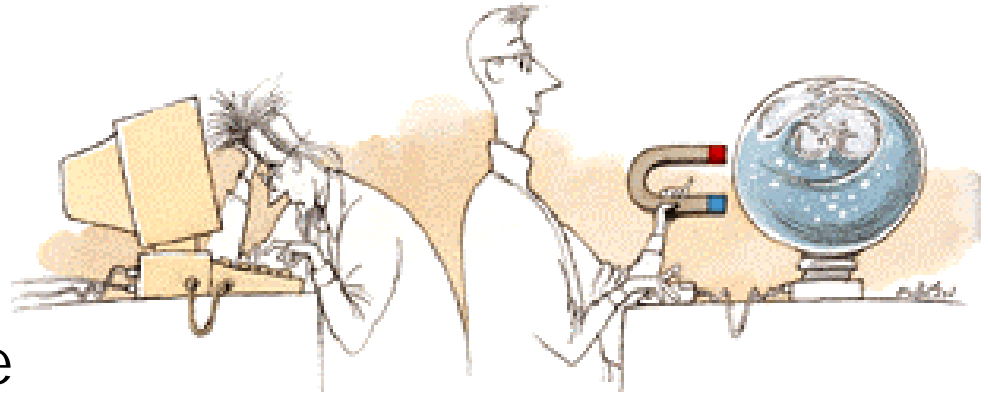
Possible solutions may exist with quantum bits



Is Quantum the Solution?

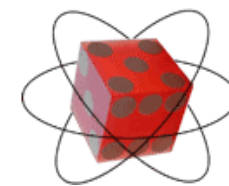
- Pros (non-classical)

- Superposition - qubits
- Entanglement - ebits
- Unitary and Reversible
- Quantum Speedup for some algorithms



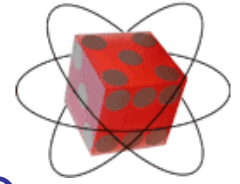
- Cons (paradigm shift)

- Distinct states not distinguishable
- Probabilistic Measurement
- Ensemble Computing and Error Correction
- Decoherence and noise
- No known scalable manufacturing process



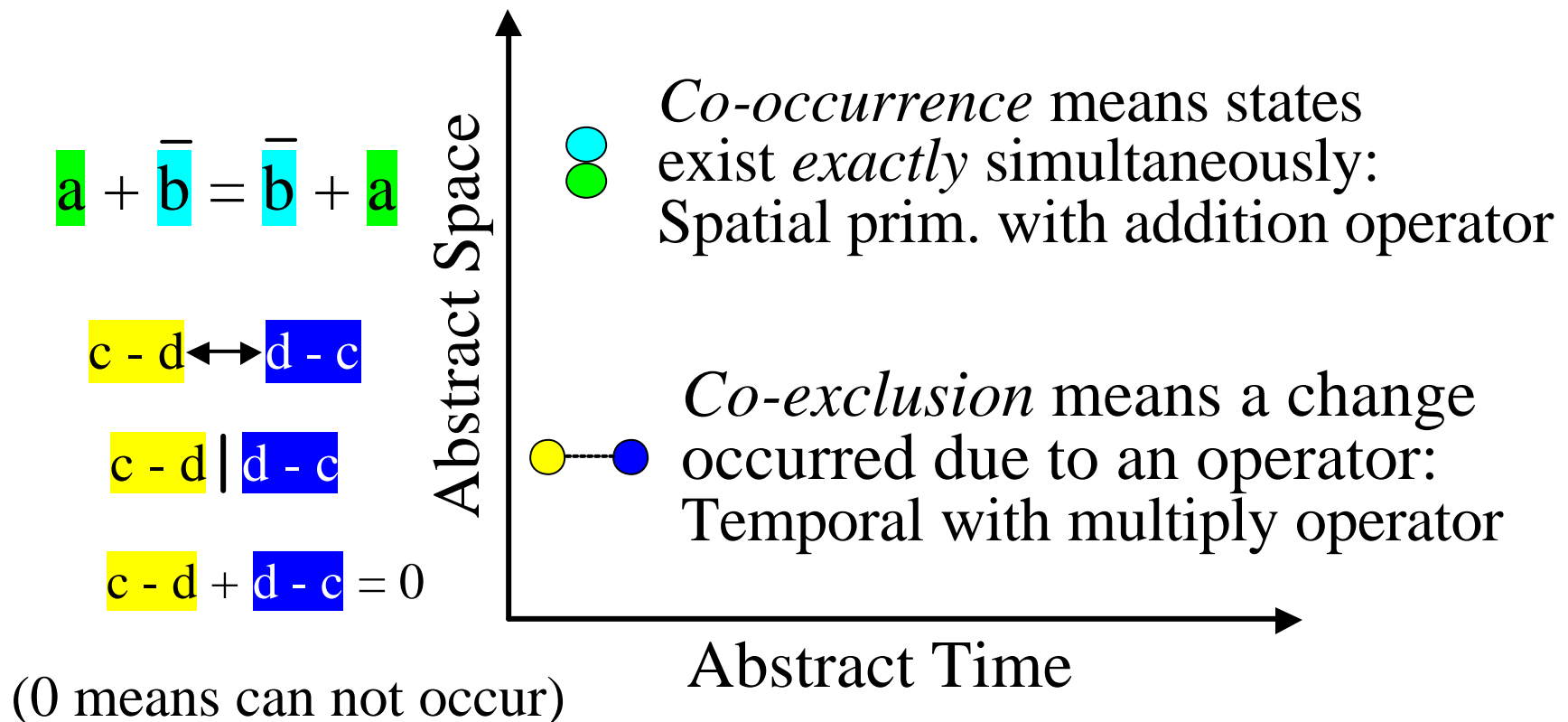
Classical vs. Quantum Bits

Topic	Classical	Quantum
Bits	Binary values 0/1	Qubits $c_0 0\rangle + c_1 1\rangle$
States	Mutually exclusive	Linearly independ.
Operators	Nand/Nor gates	Matrix Multiply
Reversibility	Toffoli/Fredkin gate	Qubits are unitary
Measurement	Deterministic	Probabilistic
Superposition	<i>Code division mlp</i>	Mixtures of $ 0\rangle$ & $ 1\rangle$
Entanglement	<i>none</i>	Ebits $c_0 00\rangle + c_1 11\rangle$

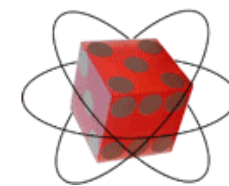


Abstract Notions of Space & Time

Co-Occurrence and Co-Exclusion

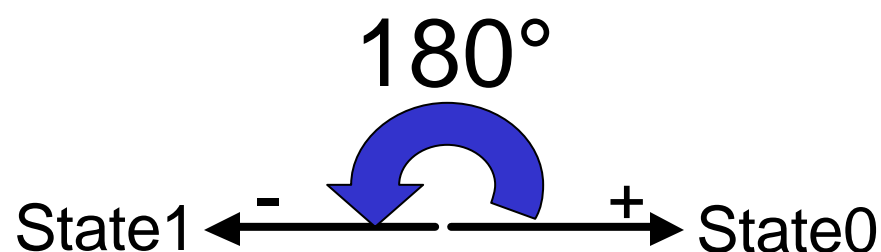


More & coin demonstration in my Ph.D dissertation



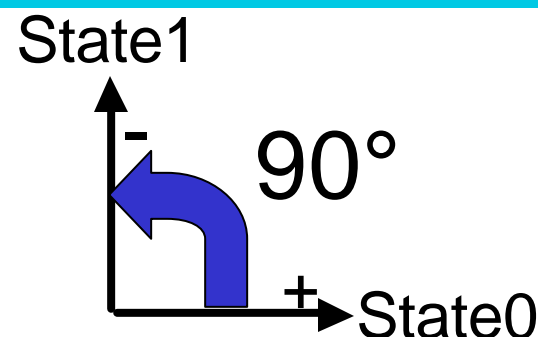
Quantum Bits – Qubits

Classical bit states:
Mutual Exclusive



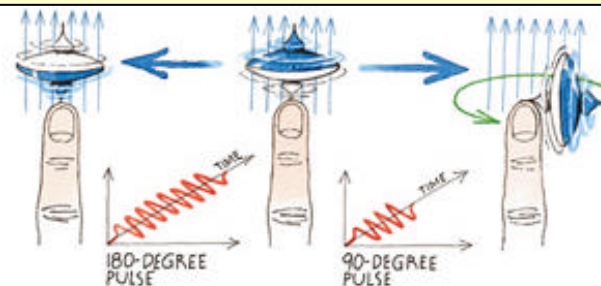
Classical states
co-exclude others

Quantum bit states:
Orthogonal

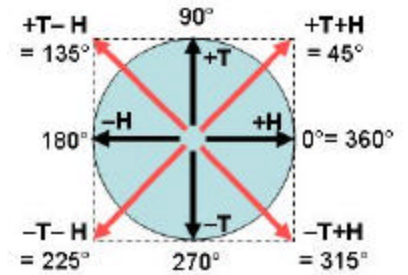


Qubits states are
called spin $\frac{1}{2}$

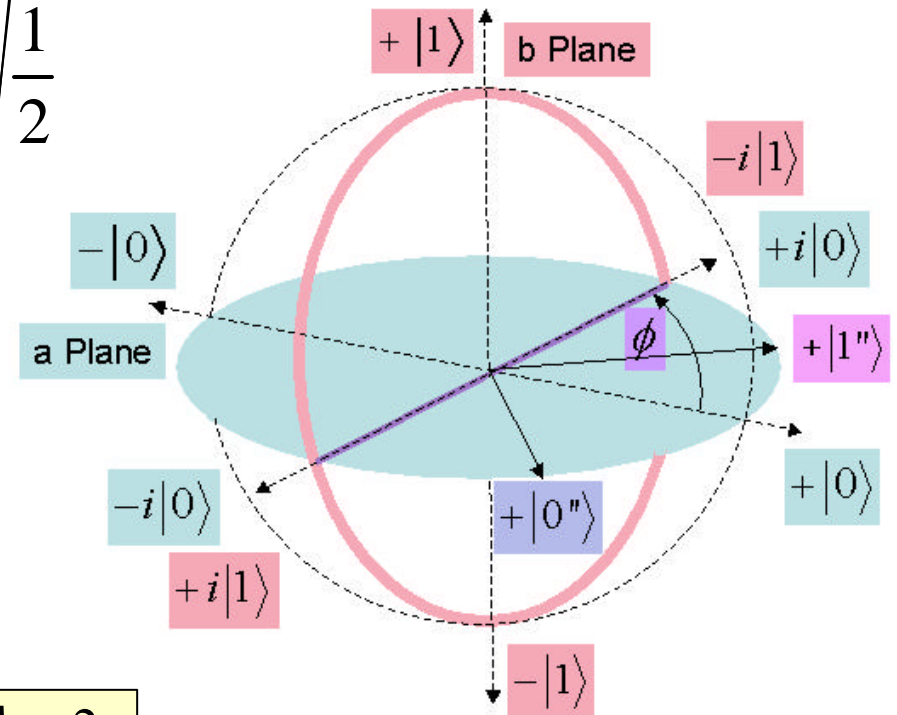
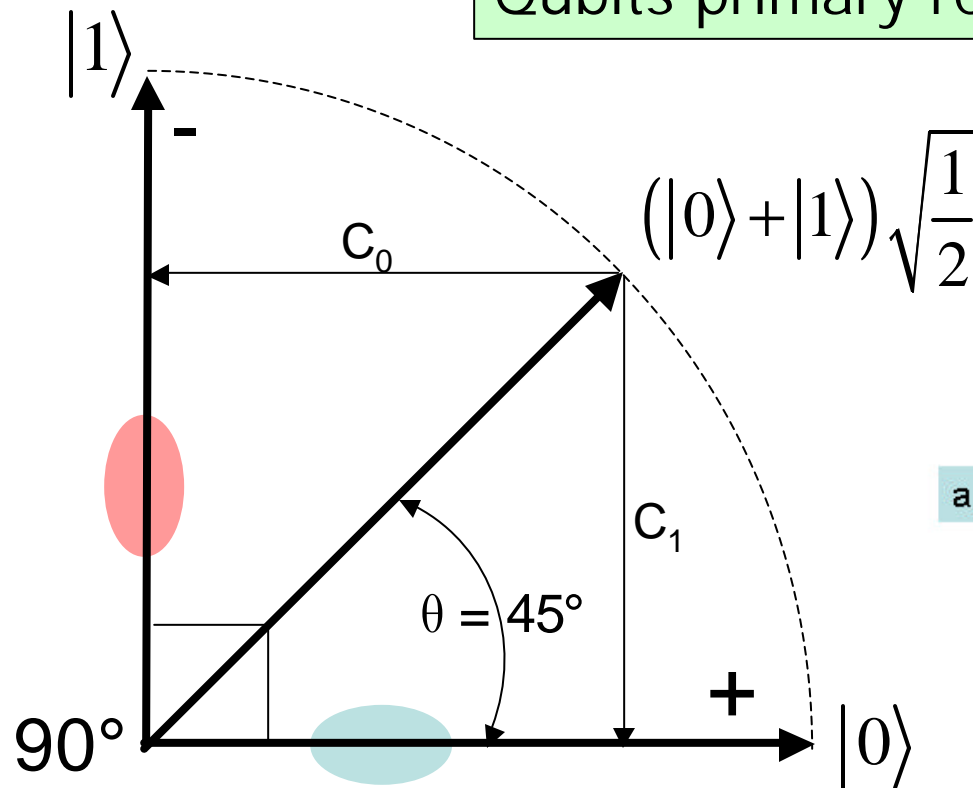
Quantum States are *orthogonal*:
not mutually exclusive!



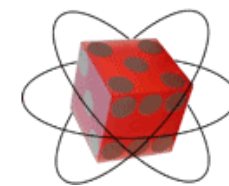
Phases & Superposition



Qubits primary representation is Phase Angle



Unitarity Constraint is $1 = \sum c_i^2$



Qubit and Ebit Details

■ Qubit

q0

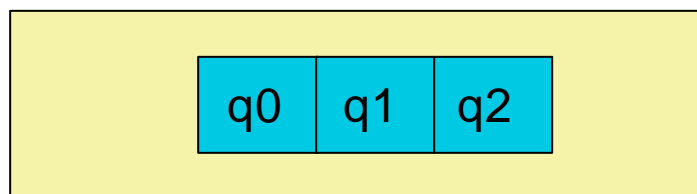
$$c_0 |0\rangle + c_1 |1\rangle$$

q1

$$c_0 |0\rangle + c_1 |1\rangle$$

not * q0
phase * q1

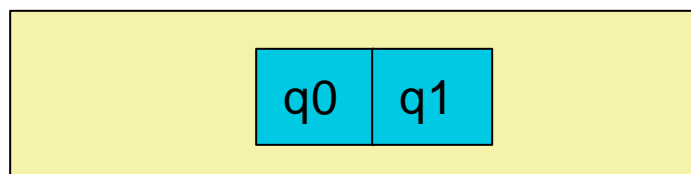
■ Qureg



$$q0 \otimes q1 \otimes q2$$

$$c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle + c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle$$

■ Ebit

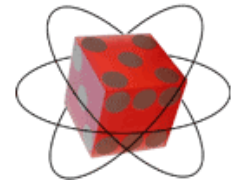


$$\text{bell}^*(q0 \otimes q1)$$

$$c_0 |00\rangle + c_1 |11\rangle \quad \text{or} \quad c_0 |01\rangle + c_1 |10\rangle$$

\otimes = tensor product

Matrices 101 (Quick Review)



$$\mathbf{S} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

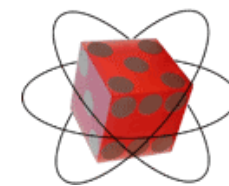
$$\Psi = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$$

$$c_0 = 1/\sqrt{2} \\ = 0.707$$

$$\mathbf{S} * \Psi = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} a\mathbf{a} + b\mathbf{b} \\ c\mathbf{a} + d\mathbf{b} \end{bmatrix}$$

$$\mathbf{S}_1 * |0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0*1 + 1*0 \\ 1*1 + 0*0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$H * |0\rangle = c_0 \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_0 \begin{bmatrix} 1*1 + 1*0 \\ 1*1 + -1*0 \end{bmatrix} = c_0 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_0 \end{bmatrix}$$



Quregister: Matrices 201

$$\begin{aligned} state0_0 &= |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ state1_0 &= |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

(tensor product) \otimes =

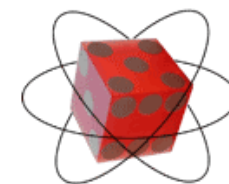
$$\begin{aligned} state0_1 &= |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ state1_1 &= |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

$$state0 = |00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad state1 = |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$state2 = |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad state3 = |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

\langle | Bra is row vector $|$ \rangle Ket is column vector

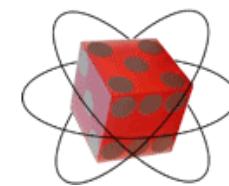
(inner product) $\langle j | * | i \rangle = \langle j | i \rangle = 0$ when $i \neq j$



Qubit Operators

Gate	Symbolic	Matrix	Circuit
Identity	$\sigma_0 * \psi$	$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	ψ —
Not (Pauli-X)	$\sigma_1 * \psi$	$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	ψ — \boxed{X} —
Shift (Pauli-Z)	$\sigma_3 * \psi$	$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	ψ — \boxed{Z} —
Rotate	$\theta * \psi$	$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$	ψ — $\boxed{\theta}$ —
Hadamard	$H * \psi$	$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	ψ — \boxed{H} —

$\begin{bmatrix} |0\rangle & |1\rangle \end{bmatrix}$

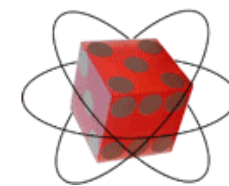


Quantum Noise

■ Pauli Spin Matrices

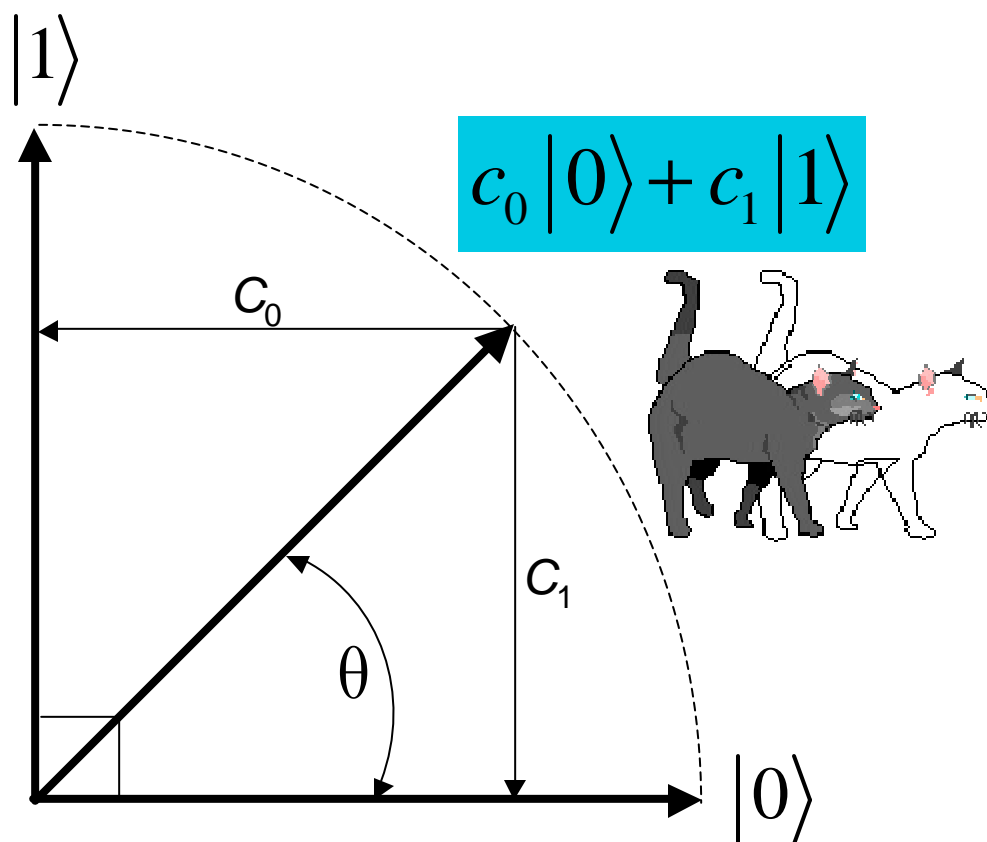
Identity	$\mathbf{s}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mathbf{s}_0 * \mathbf{y}$
Bit Flip Error	$\mathbf{s}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\mathbf{s}_1 * \mathbf{y}$
Phase Flip Error	$\mathbf{s}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\mathbf{s}_3 * \mathbf{y}$
Both Bit and Phase Flip Error	$\mathbf{s}_2 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$	$\mathbf{s}_2 * \mathbf{y}$

$$\begin{bmatrix} a & b \\ b^* & c \end{bmatrix} = \frac{1}{2}(a+d)\mathbf{s}_0 + \frac{1}{2}(b+b^*)\mathbf{s}_1 + \frac{1}{2}i(b-b^*)\mathbf{s}_2 + \frac{1}{2}(a-d)\mathbf{s}_3$$



Quantum Measurement

Probability of state $c_i |i\rangle$ is $p_i = c_i^2$ and $p_1 = 1 - p_0$

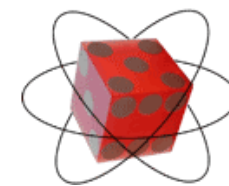


Destructive and Probabilistic!!

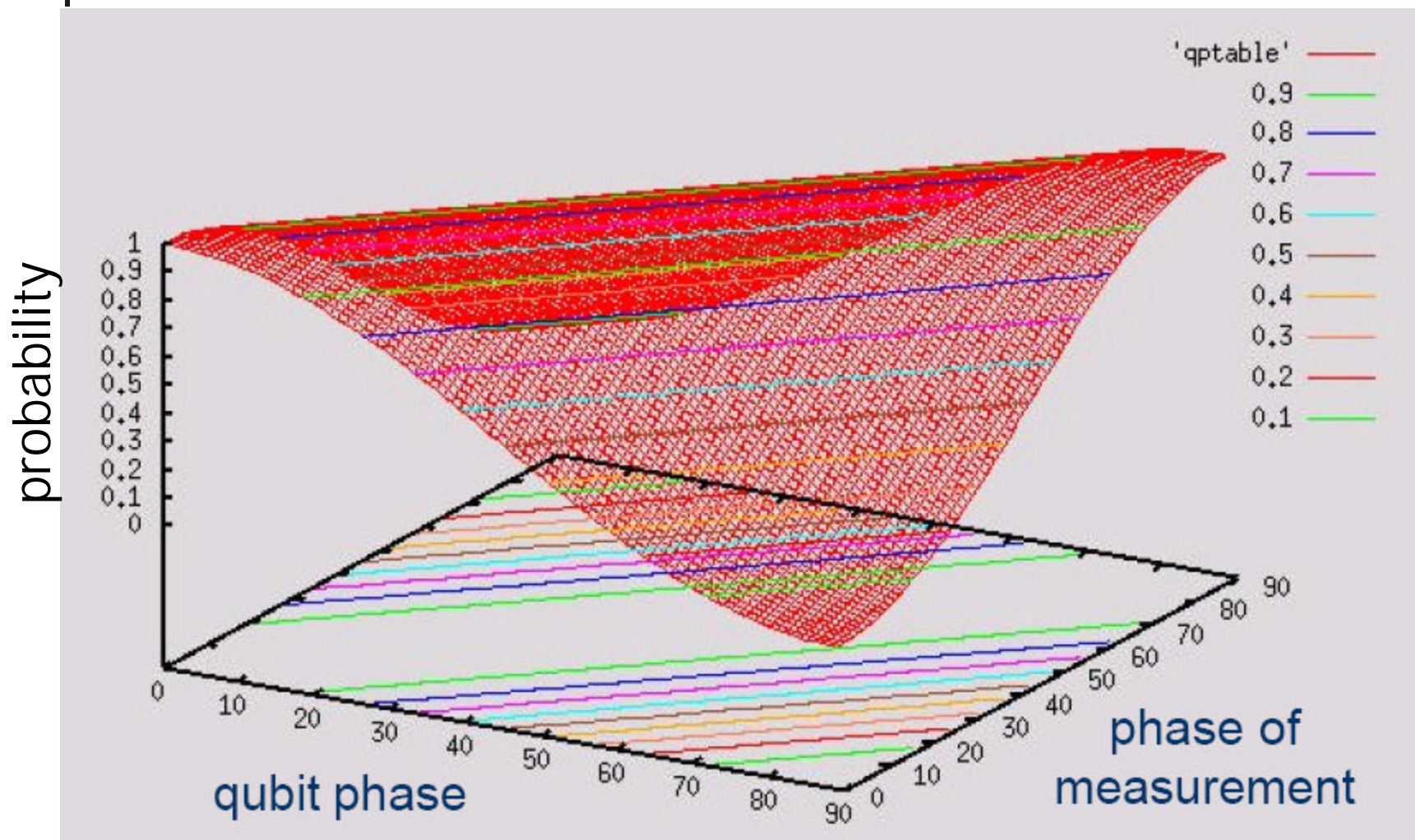
When $c_0 = c_1 = \sqrt{\frac{1}{2}}$

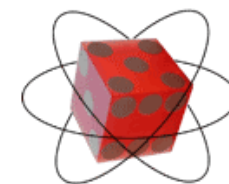
then $p_0 = p_1 = \frac{1}{2}$

or 50/50 random!



Quantum Measurement

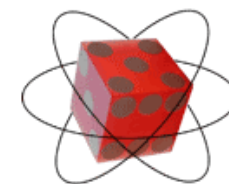




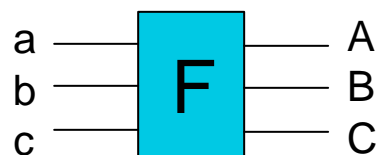
Quregisters Operators

Gate	Symbolic	Matrix	Circuit
Cnot = Control-not	$cnot * \psi$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	ψ —●— Φ —⊕—
Cnot2	$cnot2 * \psi$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	ψ —⊕— Φ —●—
swap= cnot*cnot2*cnot	$swap * \psi$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	ψ —●—⊕—●— Φ —⊕—●—⊕— =

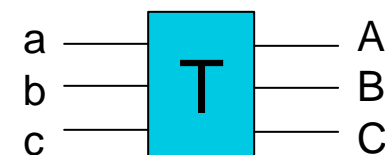
$[|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle]$



Reversible Computing



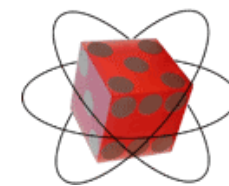
3 in & 3 out



c	b	a	C	B	A
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	1	1	1
Fredkin Gate c=control					

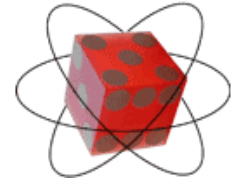
c	b	a	C	B	A
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0
Toffoli Gate c=b=control					

2 gates back-to-back gives unity gate: $T \cdot T = 1$ and $F \cdot F = 1$



Reversible Quantum Circuits

Gate	Symbolic	Matrix	Circuit
Toffoli = control-control- not	$T * \mathbf{y}$	$\begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ 0 & & & & & & 0 & 1 \\ & & & & & & 1 & 0 \end{bmatrix}$	\mathbf{y}_1 —●— \mathbf{y}_2 —●— \mathbf{y}_3 —⊕—
Fredkin = control-swap	$F * \mathbf{y}$	$\begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 0 & 1 \\ 0 & & & & & 1 & 0 \\ & & & & & & & 1 \end{bmatrix}$	\mathbf{y}_1 —●— \mathbf{y}_2 —×— \mathbf{y}_3 —×—
Deutsch	$D * \mathbf{y}$	$\begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ 0 & & & & & & i \cos q & \sin q \\ & & & & & & \sin & i \cos q \end{bmatrix}$	\mathbf{y}_1 —●— \mathbf{y}_2 —●— \mathbf{y}_3 —□—



Entangled Bits – Ebits

- EPR (Einstein, Podolski, Rosen)

- Bell States

$$B_0 = \Phi^+ = c_0 (|00\rangle + |11\rangle), \quad B_1 = \Phi^- = c_0 (|00\rangle - |11\rangle)$$

$$B_2 = \Psi^+ = c_0 (|01\rangle + |10\rangle), \quad B_3 = \Psi^- = c_0 (|01\rangle - |10\rangle)$$

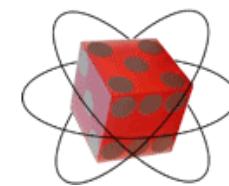
- Magic States

$$M_0 = c_0 (|00\rangle + |11\rangle), \quad M_1 = c_1 (|00\rangle - |11\rangle)$$

$$M_2 = c_1 (|01\rangle + |10\rangle), \quad M_3 = c_0 (|01\rangle - |10\rangle)$$

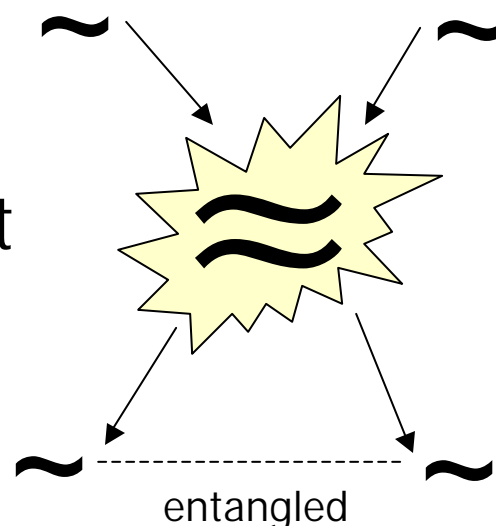
$$c_0 = 1/\sqrt{2}$$

$$c_1 = i/\sqrt{2}$$



EPR: Non-local connection

- Step1: Two qubits
- Step2: Entangle \rightarrow Ebit
- Step3: Separate
- Step4: Measure a qubit
 - Other is same if Φ^\pm
 - Other is opposite if Ψ^\pm



$$|0_0\rangle, |0_1\rangle$$

$$\Phi^\pm = |00\rangle \pm |11\rangle$$

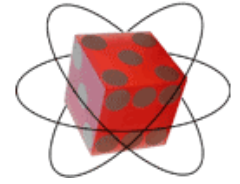
$$\Psi^\pm = |01\rangle \pm |10\rangle$$

$$|?\rangle, |?\rangle$$

answer = 1, other = 1

answer = 1, other = 0

Linked coins analogy

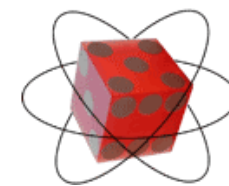


Why is quantum information special?

Quantum Computing requires a paradigm shift!!

- Quantum states are high dim (Hilbert space)
 - Can be smarter in higher dims with **no** time
 - Superposition creates new dims (tensor products)
- Quantum states are non-local in 3d & atemporal
 - Causality and determinacy are not the primary ideas
 - Large scale unitary consistency constraint system

Quantum information precedes space/time
and energy/matter - Wheeler's "It from Bit"

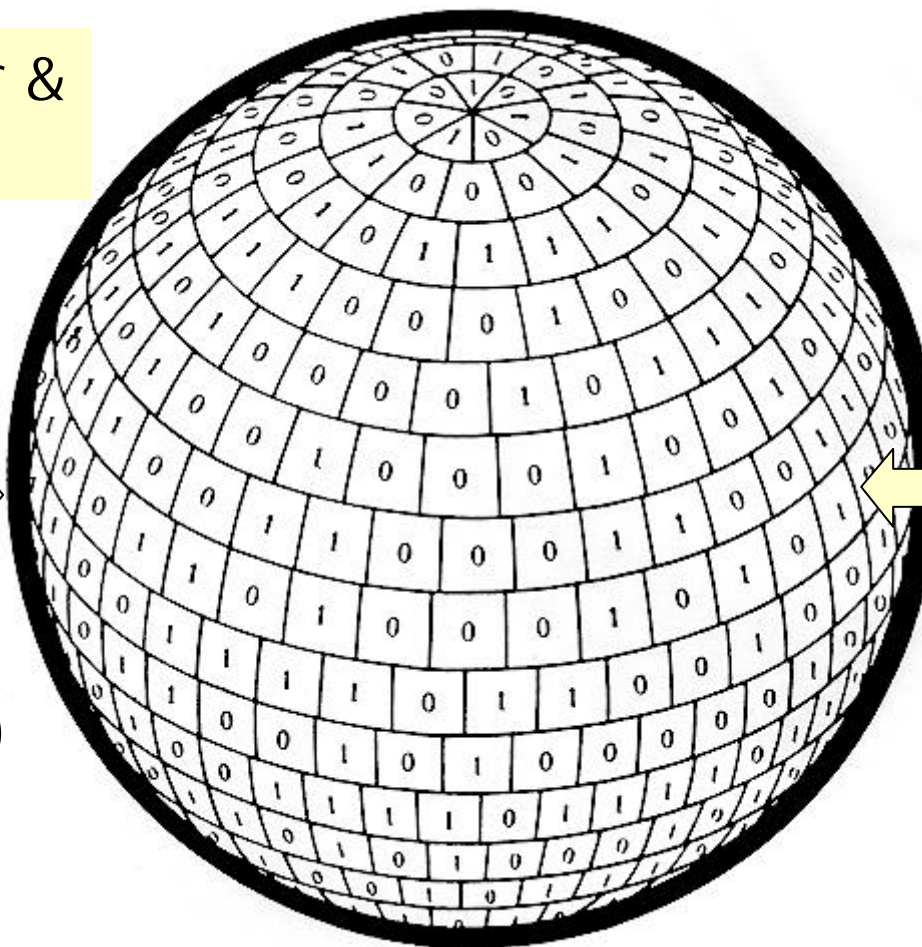


Information is Physical

Rolf Landauer &
phase spaces

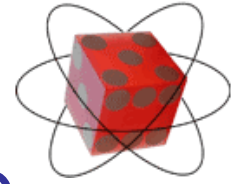
Quantum
Information is
consistent with
Black Hole
Mechanics

Black Hole
event horizon
(inside is a
singularity)



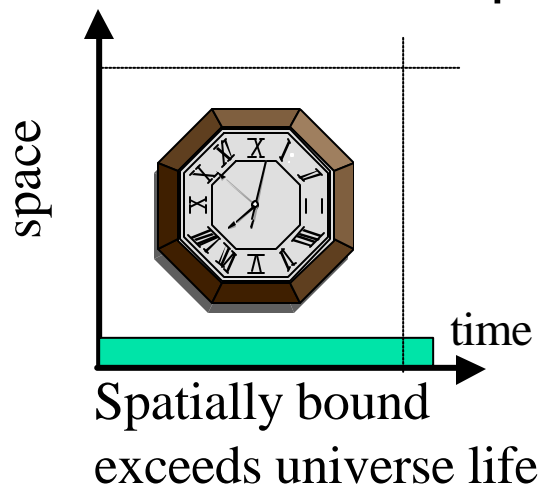
Bits as
entropy
(Planck's
areas on
surface)

Wheeler's "It from Bit"

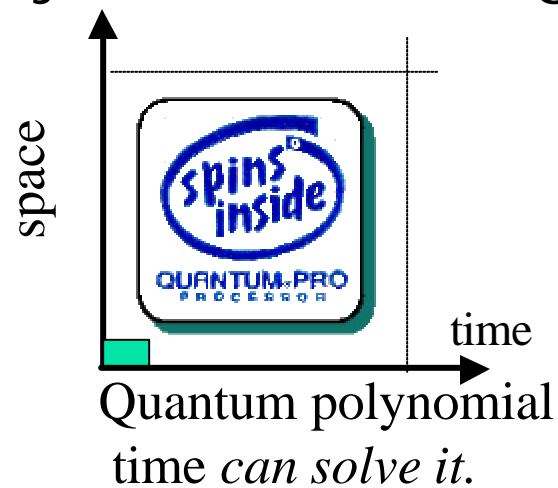


Quantum Computing Speedup

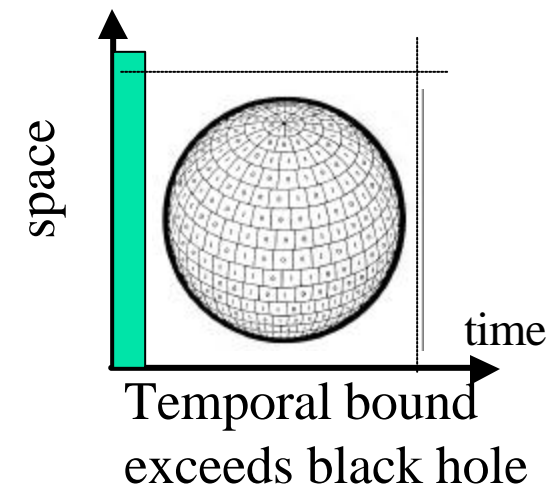
- Peter Shor's Algorithm in 1994
- Quantum Fourier Transform for factoring primes
- Quantum polynomial time algorithm



classical

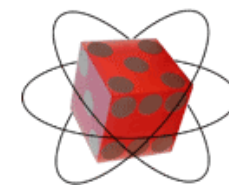


quantum



classical

Solutions to some problems don't fit in classical universe!!



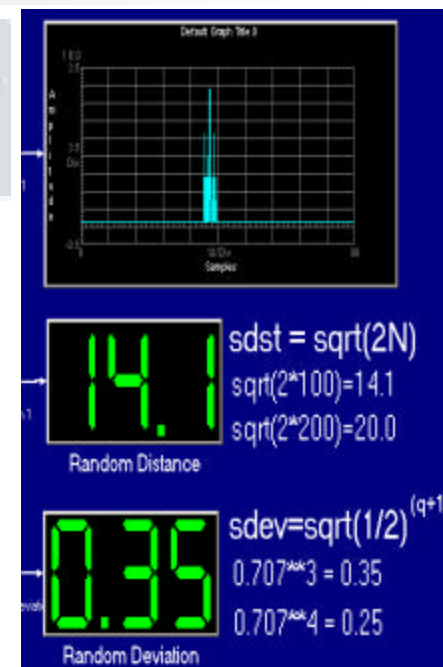
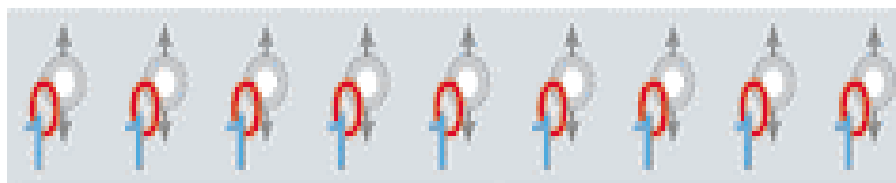
Ensemble Computing

■ Ensemble

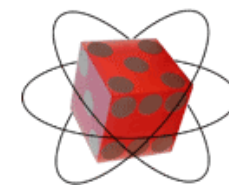
- A set of “like” things
- States can be all the same or all random!!

■ Examples

- Neurons: pulse rate
- Photons: phase angle
- Qubits: used in NMR quantum computing
- Kanerva Mems: Numenta, On Cognition, Jeff Hawkins
- Correlithm Objects: Lawrence Technologies

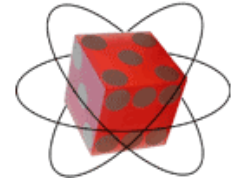


Ensembles can use randomness as a resource.



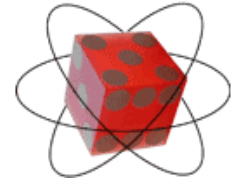
Computing Paradoxes

<i>Property</i>	<i>Choices</i>	<i>Contradiction</i>
Size	Larger/Smaller	Larger is less localized
Speed	Faster/Slower	Faster is more localized
Power	Less/more	Less power is slower
Grain Size	Gates/wires	No distinction at quantum level
Dimensions	More/less	Physical vs. mathematical dims
Parallelism	Coarse/fine	Sequential vs. Concurrent
Complexity	Less/More	Makes programming hard
Noise	Less/More	Use noise as resource
Velocity	Fast/Slow	Time Dilation slows computing



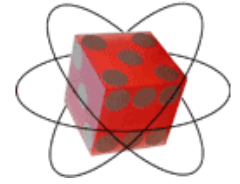
Computing Myths

- Quantum/Neural/DNA don't solve scaling
 - Quantum only applied to gate level
 - Not generalized computing systems – niches
 - Nano-computers (nanites) are science fiction
- Smarter Computers? What is Genius?
 - No generalized learning – Failure of AI
 - No general parallel computing solutions
 - Computers don't *know* anything (only data)
 - Computers don't *understand* (speech&image)
 - Computers have no *meaning* (common sense)



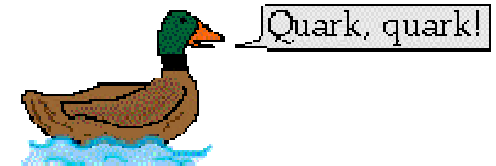
What is Genius?

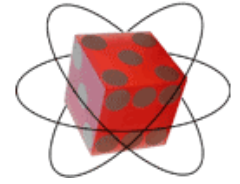
- Single Cells
 - Virus, Ameba, paramecium, neurons, jelly fish, etc
- Insects
 - Motion, sight, flying, group activity
- Small Children
 - Learning by example, abstraction
 - Motion, walking, running, emotions
 - Image and speech understanding, talking
 - Languages, music, mathematics, etc
 - Accommodation, design, planning
- Deep Blue – Chess??
 - No understanding, no meaning, no insight



Business Predictions

- Semiconductors will stop scaling in ~10 yrs
 - Nanocomputers won't stop this; only delay it
 - Breakthrough required or industry stagnates
 - College students consider non-semiconductor careers
- Research needed in these areas:
 - Deep meaning and automatic learning
 - Programming probabilistic parallel computers
 - Noise as valued resource instead of unwanted
 - Higher dimensional computing
 - Investigate non-local computing
 - Biological inspired computing – Quantum Brain?

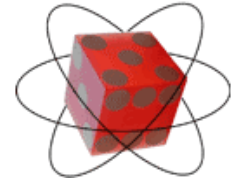




Conclusions

- Computer scaling creates uncertainty
- Quantum Computing not yet a solution
- Watch for unexpected aspects of noise
- Industry is not open on scaling problems
- Research money is lacking
- Costs may slow before limits
- Must think outside 3d box
- Focus on Human Acceleration





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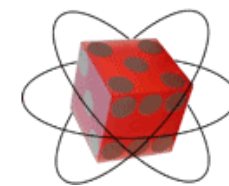
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Quantum Ensemble Example

