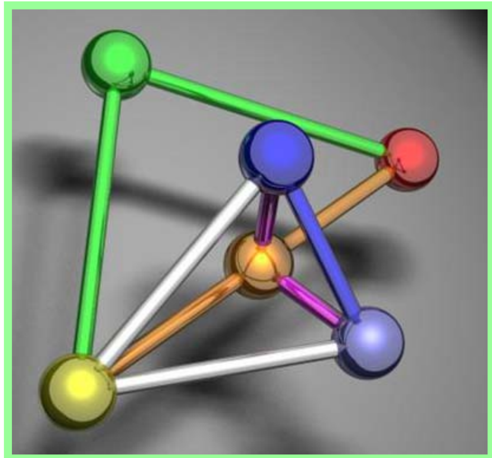
Invariant Quantum Ensemble Metrics



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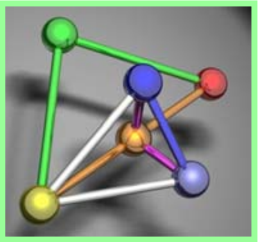
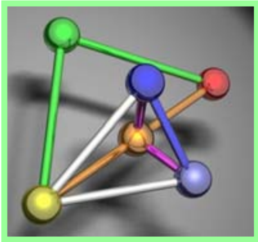
Outline of Talk



- Historical background
- Ensembles and Correlithm Objects (COs)
  - Standard distance and radius metrics
  - Normalized metrics
- Quantum ensembles
- QuCOs survive measurement
- Ensemble and quantum toolsuite
- Conclusions

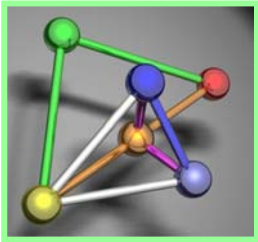
Historical Background



- Wootters (Phys Rev D Vol 23, 1981):
  - “Statistical distance and Hilbert Space”,
  - Statistical dist same as angle between states
- Levitin, Toffoli (LANL qu-ph/0122075, 2001):
  - “Information and Distinguishability of Ensembles of Identical States”
  - Number of distinguishable states related to angle and size of ensemble N.

Both use ensembles of identically prepared qubits

Correlithm Objects

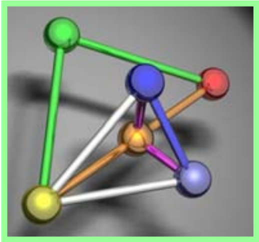


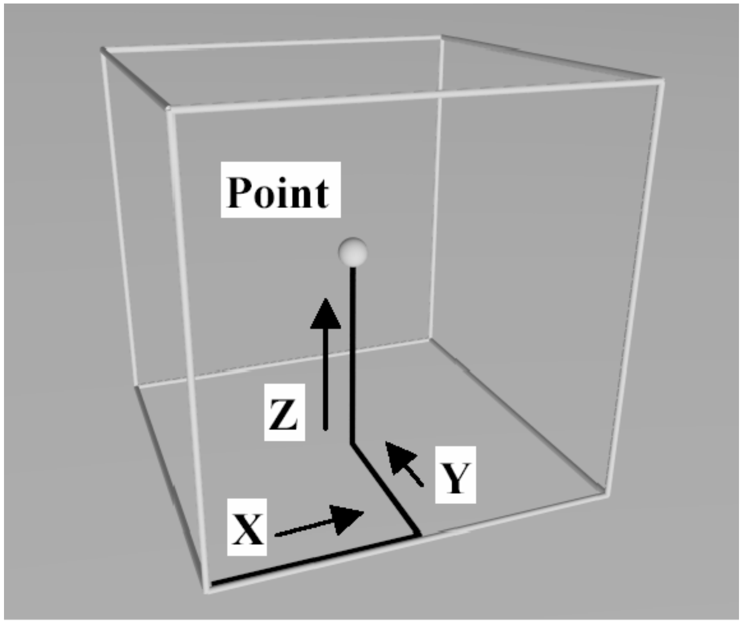
Randomly select elements of ensemble!

- Points in a Space (Unit cube, Hilbert Space)
- Cartesian Distance between Points
  - Same for all random points/corners of space
    - *Standard* Distance, *Standard* Radius and other metrics
    - Related to field of probabilistic geometry
  - Follows a Gaussian Distribution
    - Mean: grows as $\sqrt{N}$
    - Standard deviation: independent of N
  - BOOK: "Correlithm Object Technology", 2004

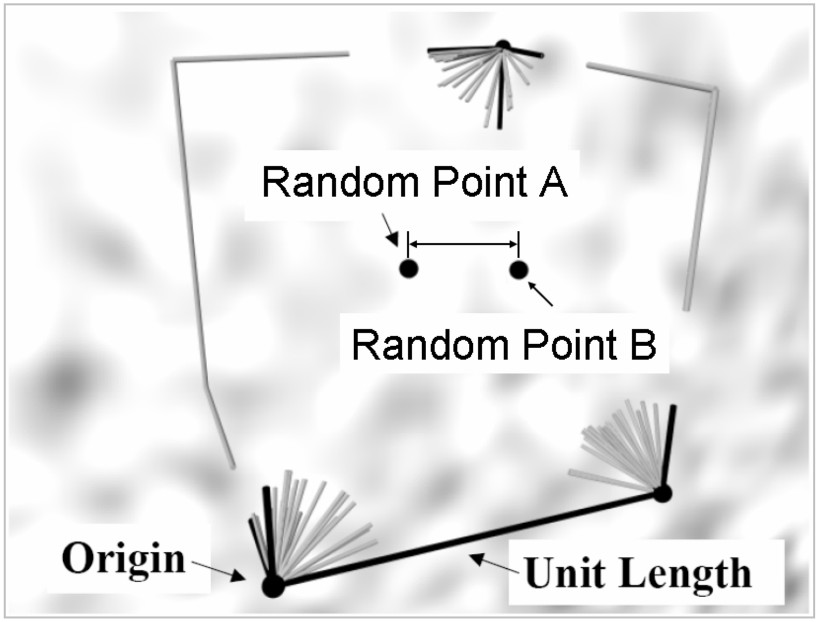
Correlithm is contraction for *Correl*ational Algor*ithm*

Correlithm Objects (COs) are Points





1 point in 3 dimensions



2 points in N dimensions

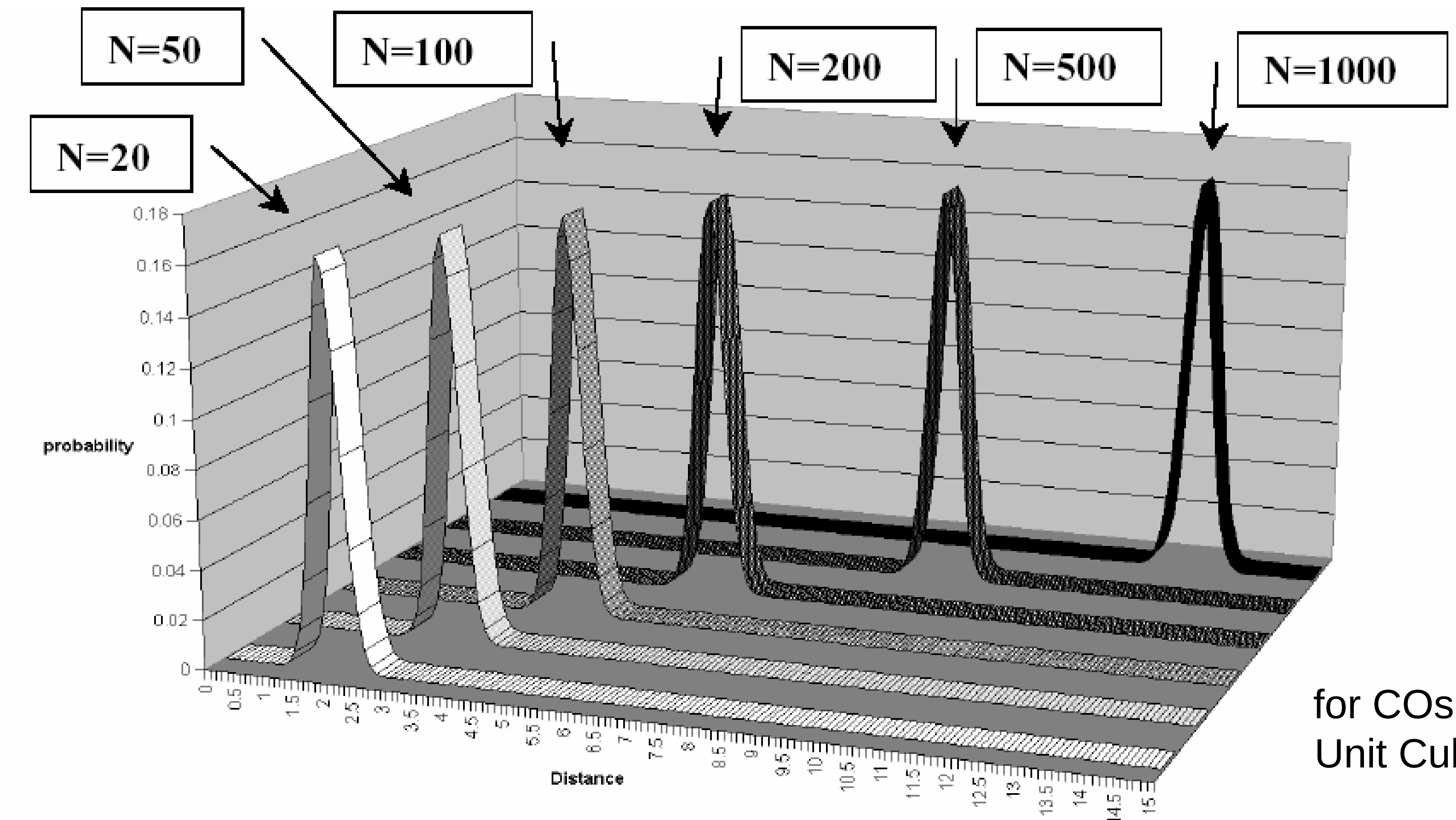
$$for\ X = [x_1, \ldots, x_N]\ and\ Y = [y_1, \ldots, y_N]$$

$$dist(X, Y) = \sqrt{(x_1 - y_1)^2 + \ldots + (x_N - y_N)^2}$$

Randomly chosen points are *standard distance* apart!

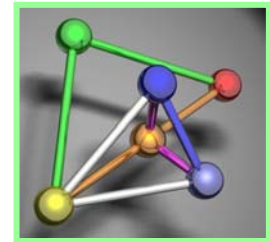
Cartesian Distance Histograms

“Standard” Distance = $\sqrt{\frac{N}{6}}$ Standard Deviation = $\sqrt{\frac{7}{120}}$



for COs in Unit Cube

Bounds on distance metric steps

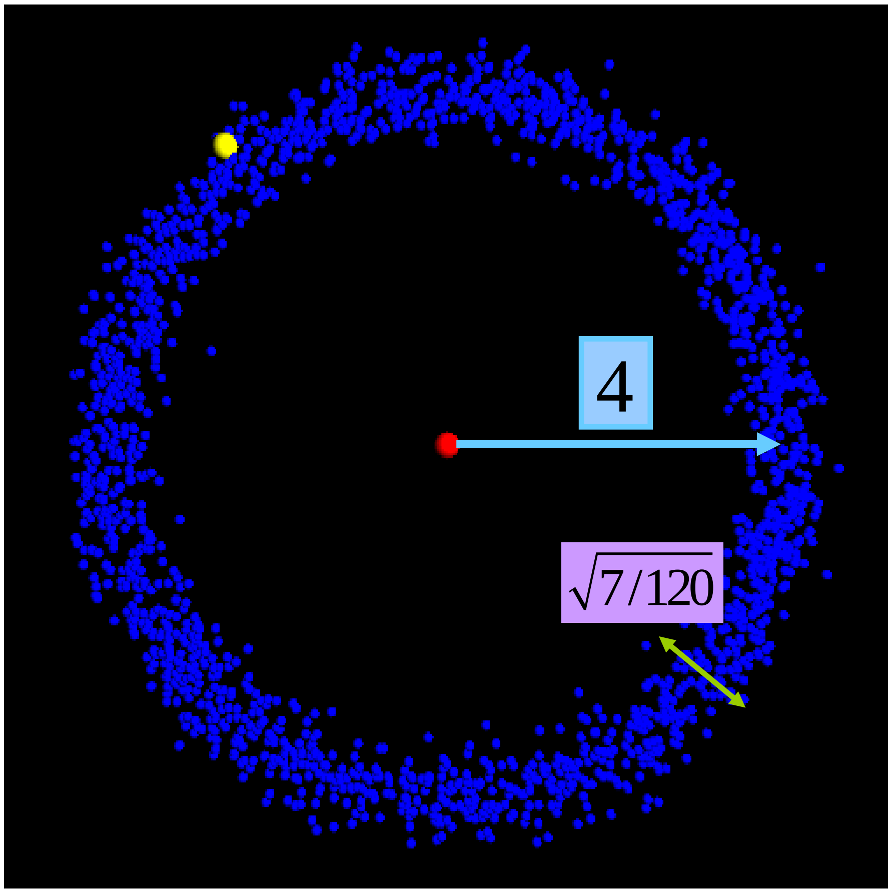


Derived by computing examples with N=1,000,000

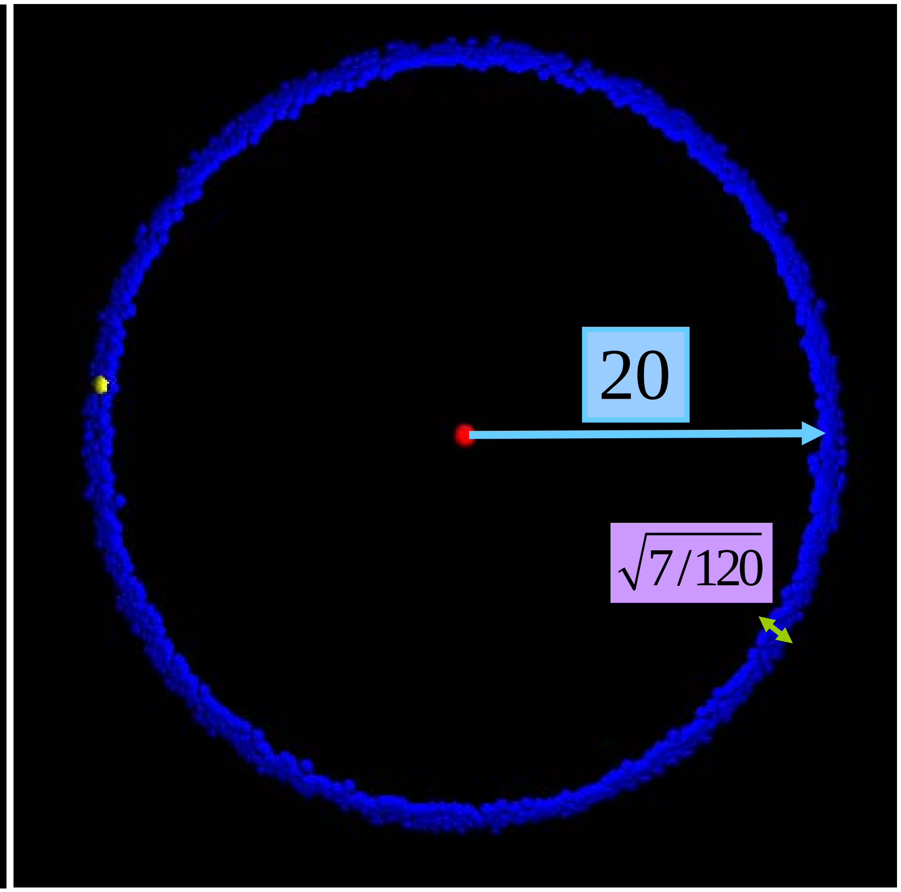
Step	Expression	Distribution for real values 0→1			
		lower	upper	average	distribution
1	a_i, b_i	0	1	≈ 1/2	uniform
2	$x_i = a_i - b_i$	–1	1	≈ 0.0	not uniform
3	$\lvert x_i \rvert$	0	1	≈ 1/3	not uniform
4	$y_i = x_i^2$	0	1	≈ 1/6	not uniform
5	$z = \sum y_i$	≈ N/6			single value with standard deviation
6	$d = \sqrt{z}$	Cartesian distance(A, B) $\approx \sqrt{N/6}$			
7	$e = z/N$	Expected value or Avg of y_i ≈ 1/6			

Constant Standard Deviation

For N=96, Standard distance = 4



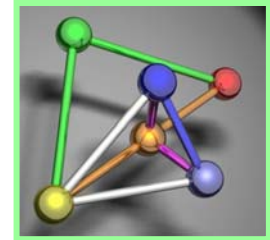
For N=2400, Standard distance = 20



Two plots are scaled/normalized to same relative size

for Unit Cube COs

Standard Distance Metrics



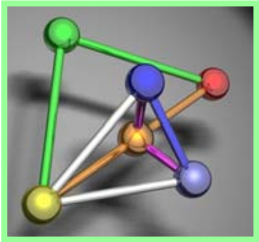
Property	Type	Cartesian Distance	Standard Deviation	Cartesian Distance	Standard Deviation
		Unit Edge		*Unit Radius*	
Edge: length of side	Exact	1	0	$\approx \sqrt{12/N}$	≈ 0
Major diagonal: max corner to corner	Exact	$\sqrt{N}$	0	$\approx \sqrt{12}$	≈ 0
R corner to R corner	Prob	$\approx \sqrt{N/2}$	$\approx \sqrt{1/9}$	$\approx \sqrt{6}$	$\approx \sqrt{4/3N}$
Standard diameter: R corner to R point	Prob	$\approx \sqrt{N/3}$	$\approx \sqrt{1/15}$	$\approx \sqrt{4}$	$\approx \sqrt{4/5N}$
Half diagonal: Half major diagonal	Exact	$\sqrt{N/4}$	0	$\approx \sqrt{3}$	≈ 0
Standard distance: R point to R point	Prob	$\approx \sqrt{N/6}$	$\approx \sqrt{7/120}$	$\approx \sqrt{2}$	$\approx \sqrt{7/10N}$
Standard Radius: Midpoint to R point	Prob	$\approx \sqrt{N/12}$	$\approx \sqrt{1/60}$	1	$\approx \sqrt{1/5N}$

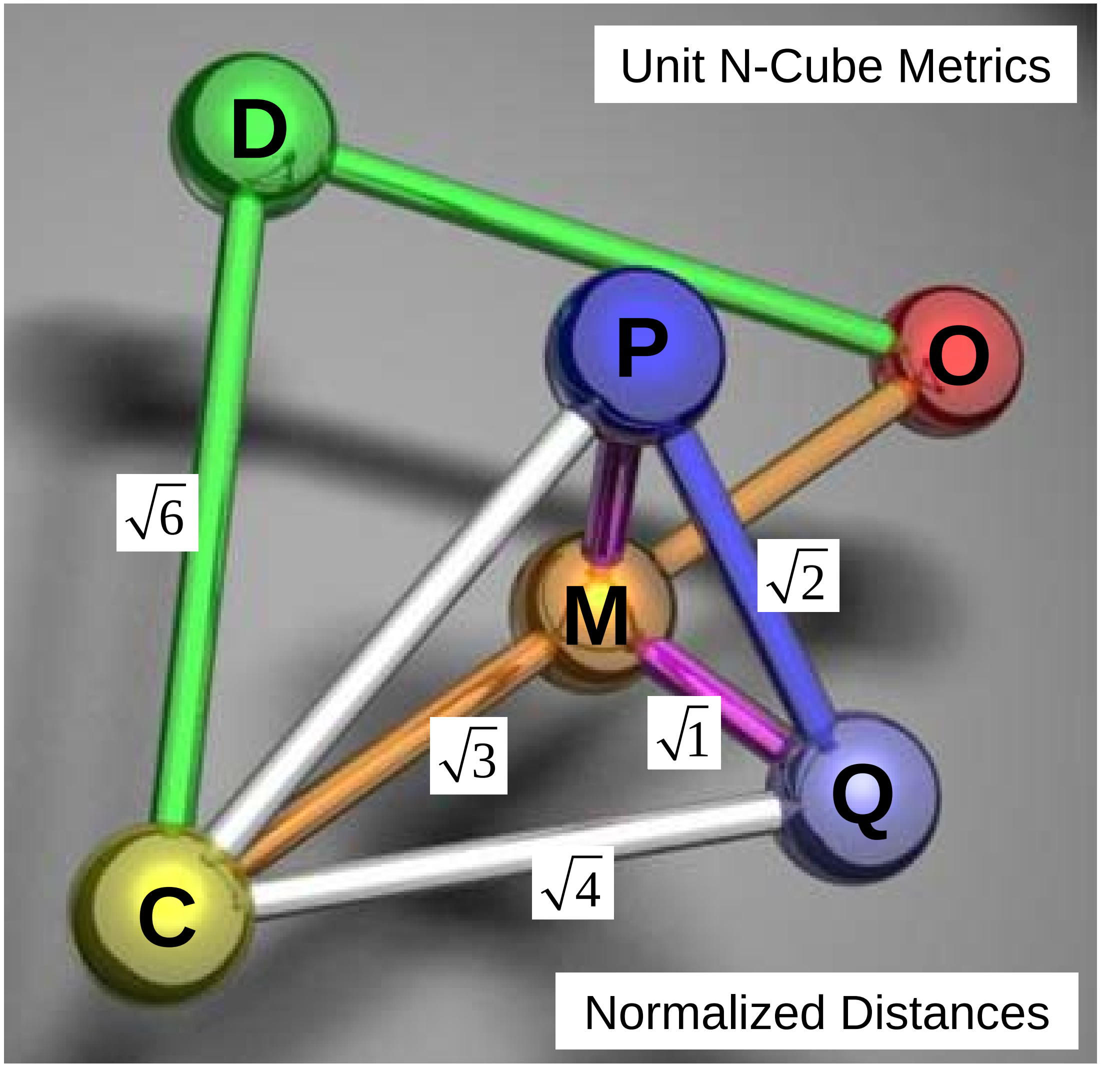
Normalized by Radius

Statistics are for random points/corners with Unit Cube COs

Equihedron Topology

Probabilistically forms high dimensional tetrahedron





Exact Points

C = Corner Reference

M = Mid point of space

O = Opposite Corner

Random Points

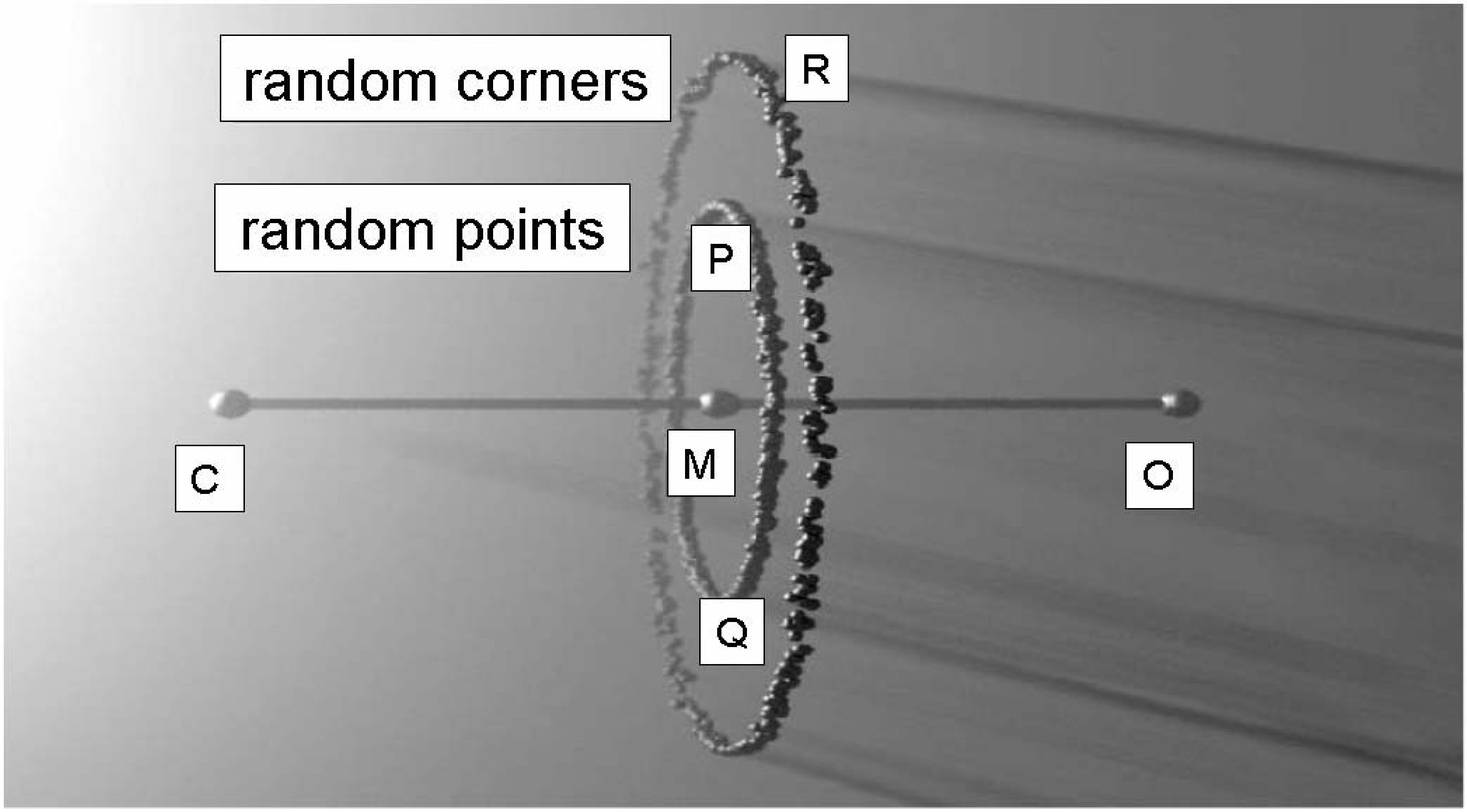
P = Random CO 1

Q = Random CO 2

D = Random Corner

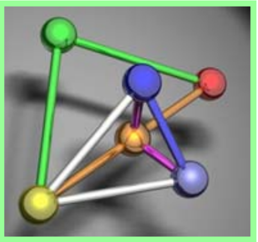
Angles are: PMQ~90°, PCM=QCM~30°, PCQ~41.4°

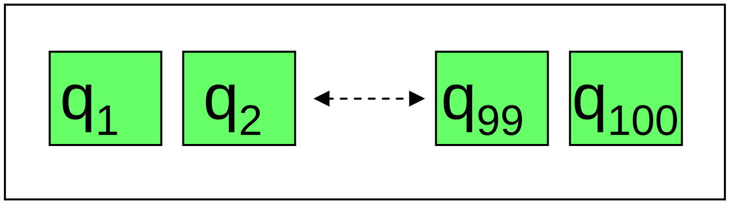
Invariant Metrics



All random CO points are *equidistant* from each other and
all random CO points are *equidistant* from center point and
all random CO corners are *equidistant* from each other …

Quantum Ensembles

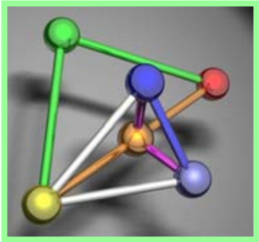


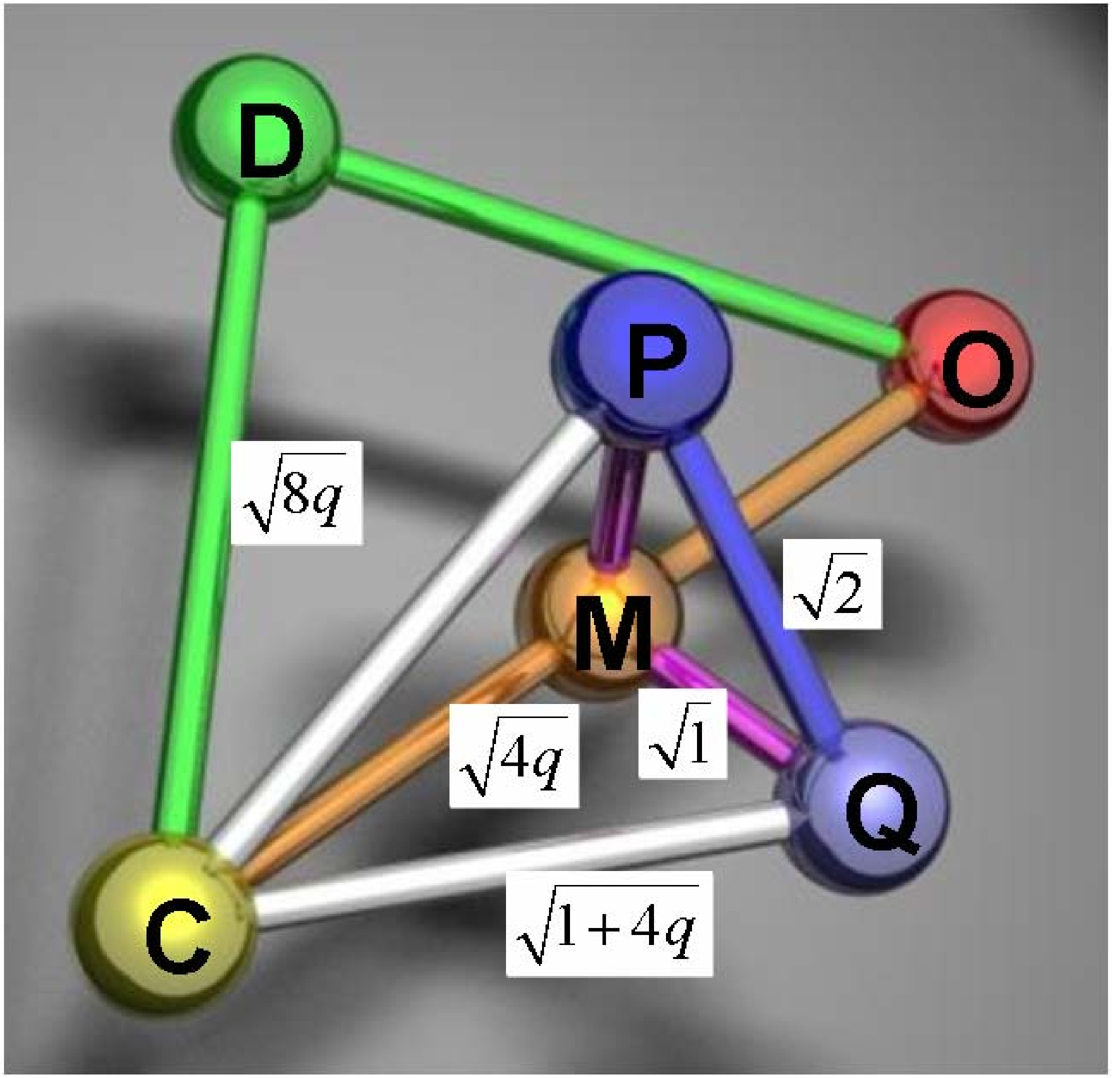


Qubit ensemble Q
of size N=100

- N qubits that are arrayed but not entangled
- If random phase for each qubit:
  - Represents a point in high dimensional space
  - Phase Invariant (all phases present)
  - Distance between two random ensembles $\sqrt{2N}$
  - Standard deviation is $\sqrt{1/2}^{q+1}$
  - Same metrics if each N is a quantum register

Quantum Ensemble Metrics





q = Number of qubits in each cell

Exact Points

C = Corner Reference

M = Mid point of space

O = Opposite Corner

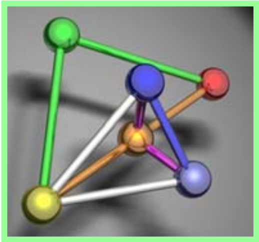
Random Points

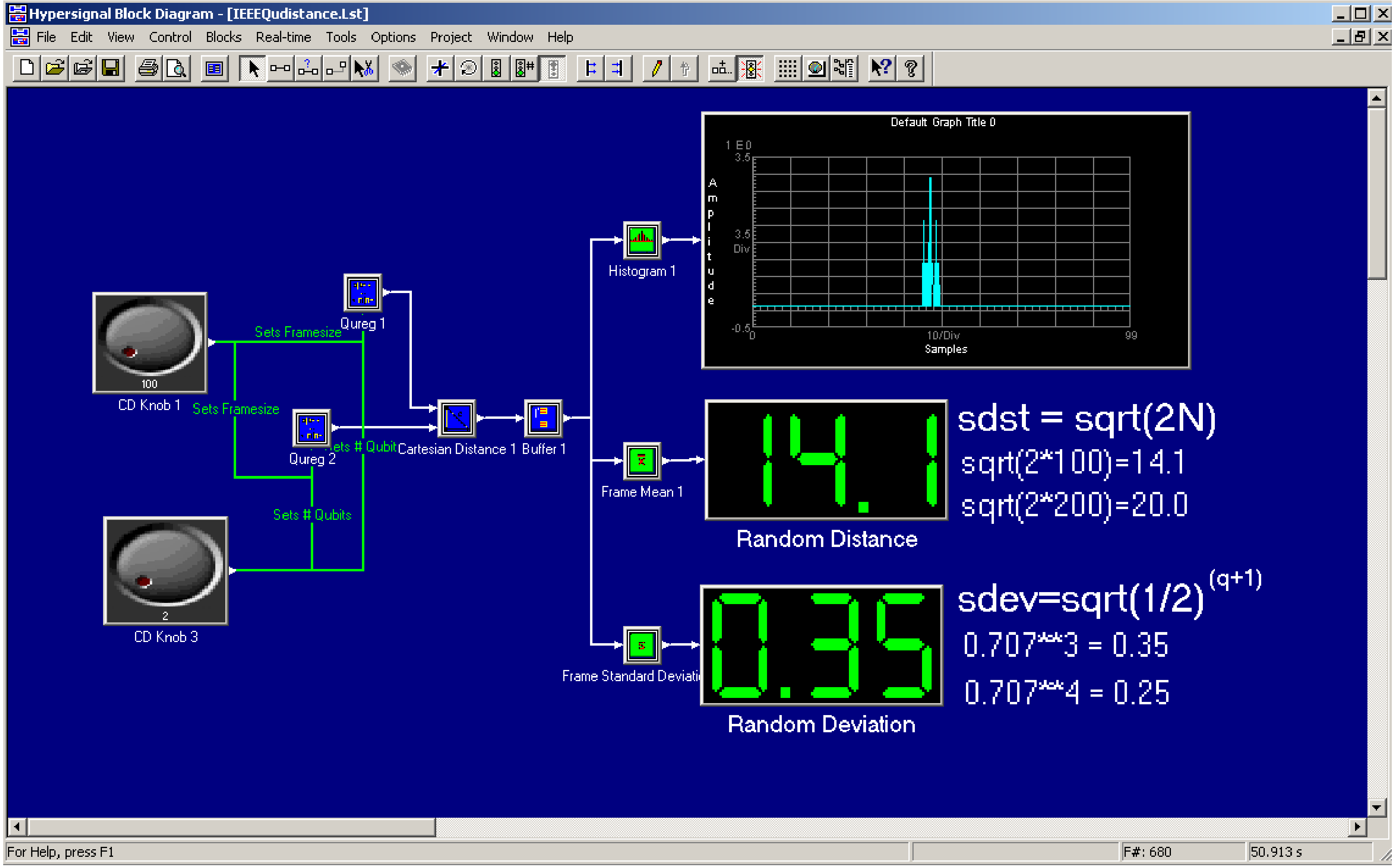
P = Random CO 1

Q = Random CO 2

D = Random Corner

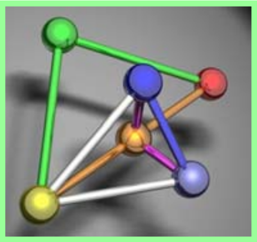
Standard Distance for QuEnsembles





Standard deviation is a independent of N

Accessing Quantum COs

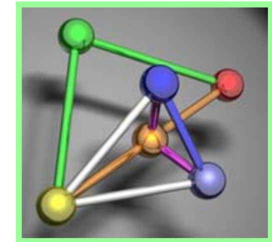


- Quantum COs are not directly visible
  - except thru simulation or measurement
- Measure of QuCOs produces classical cell CO
  - Answer is binary CO
  - End state is another QuCO
- Multiple trials reveals underlying QuCO

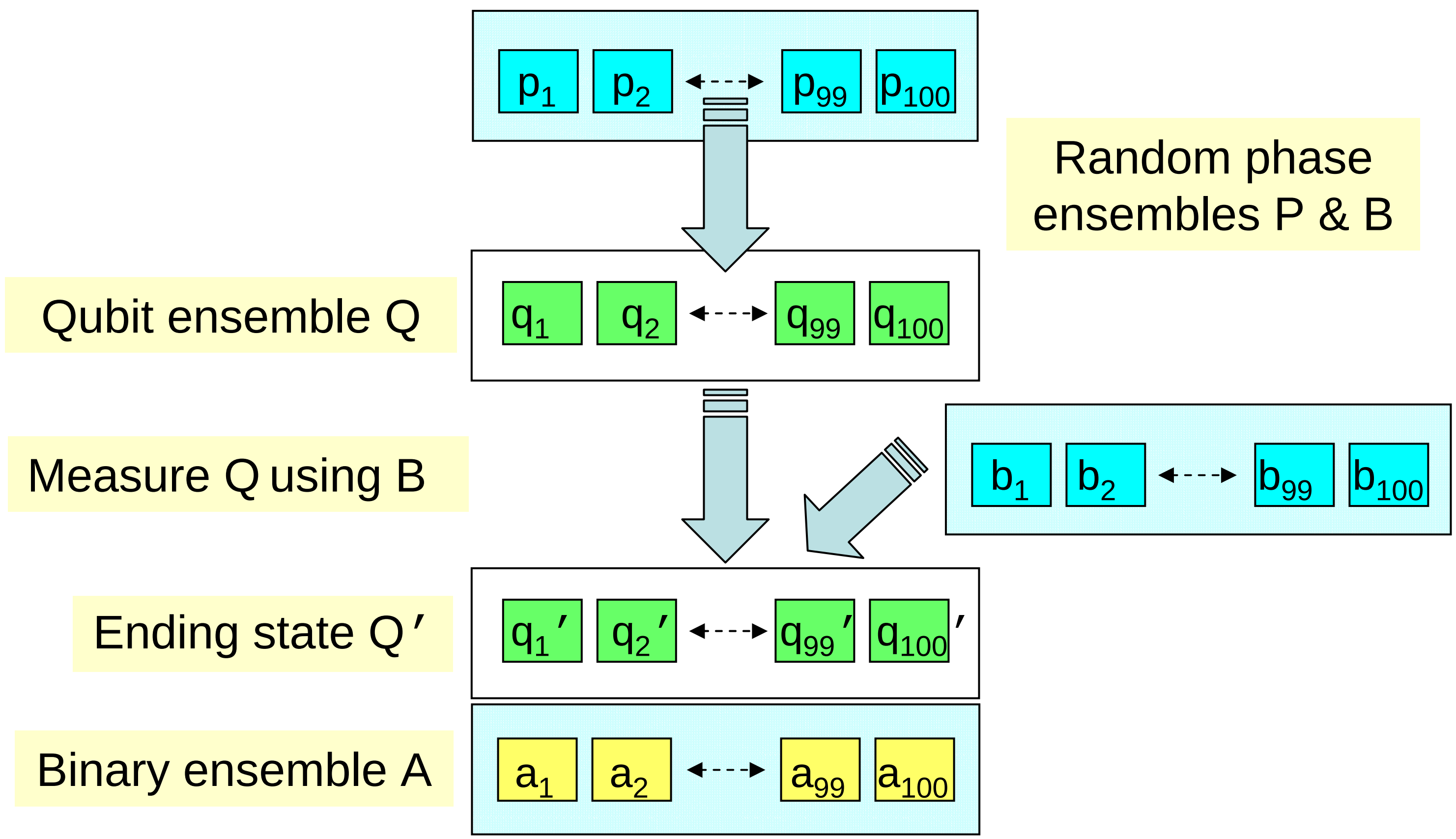
Measurement acts as noise injection CO process!

CO tokens survive this process!

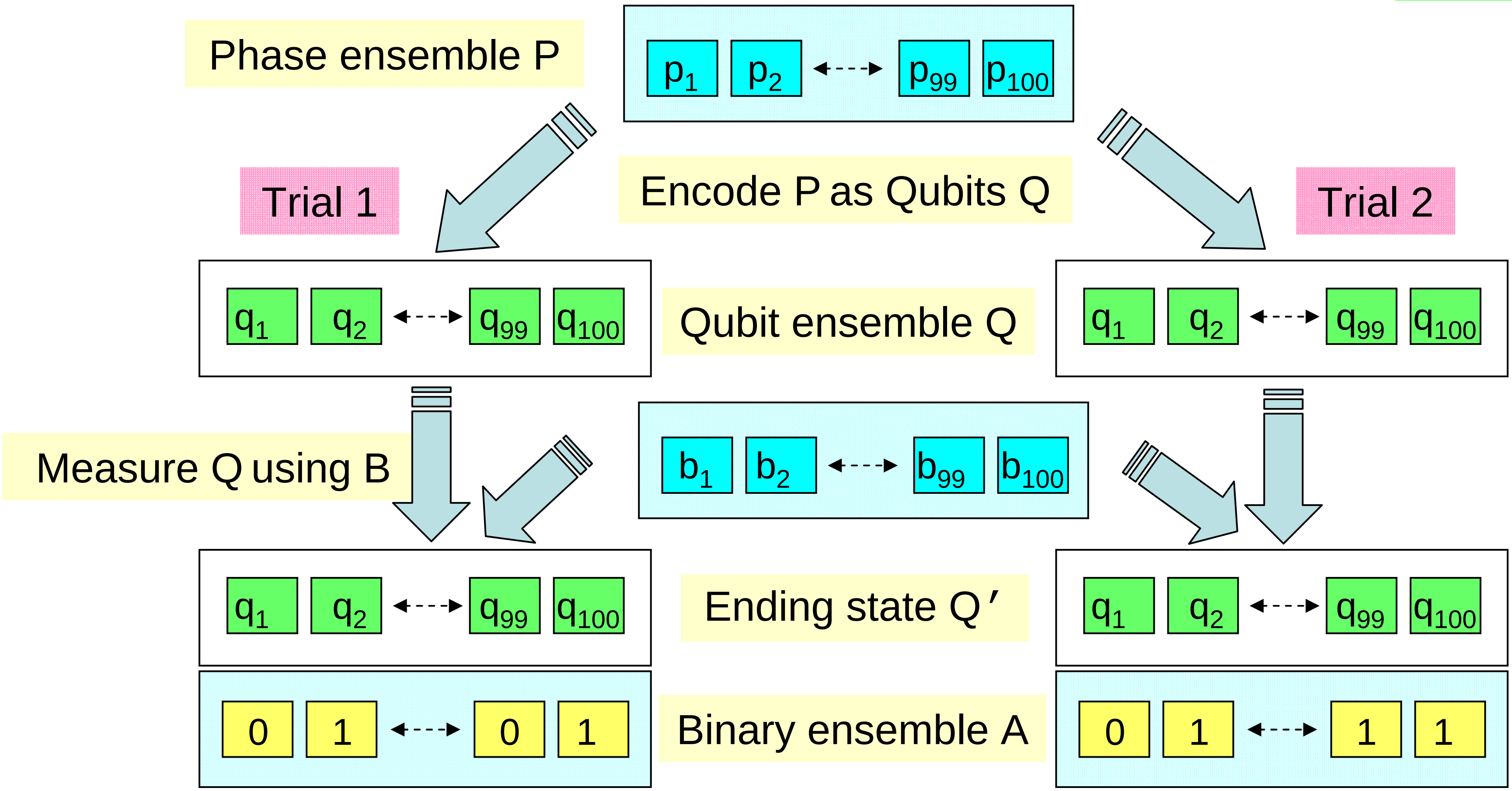
Quantum CO measurement



Uses Phase Ensembles, Qubit Ensembles and Binary Ensembles



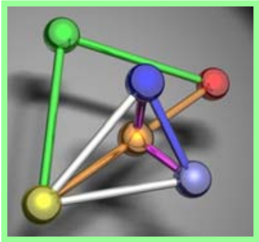
COs Survive Measurement

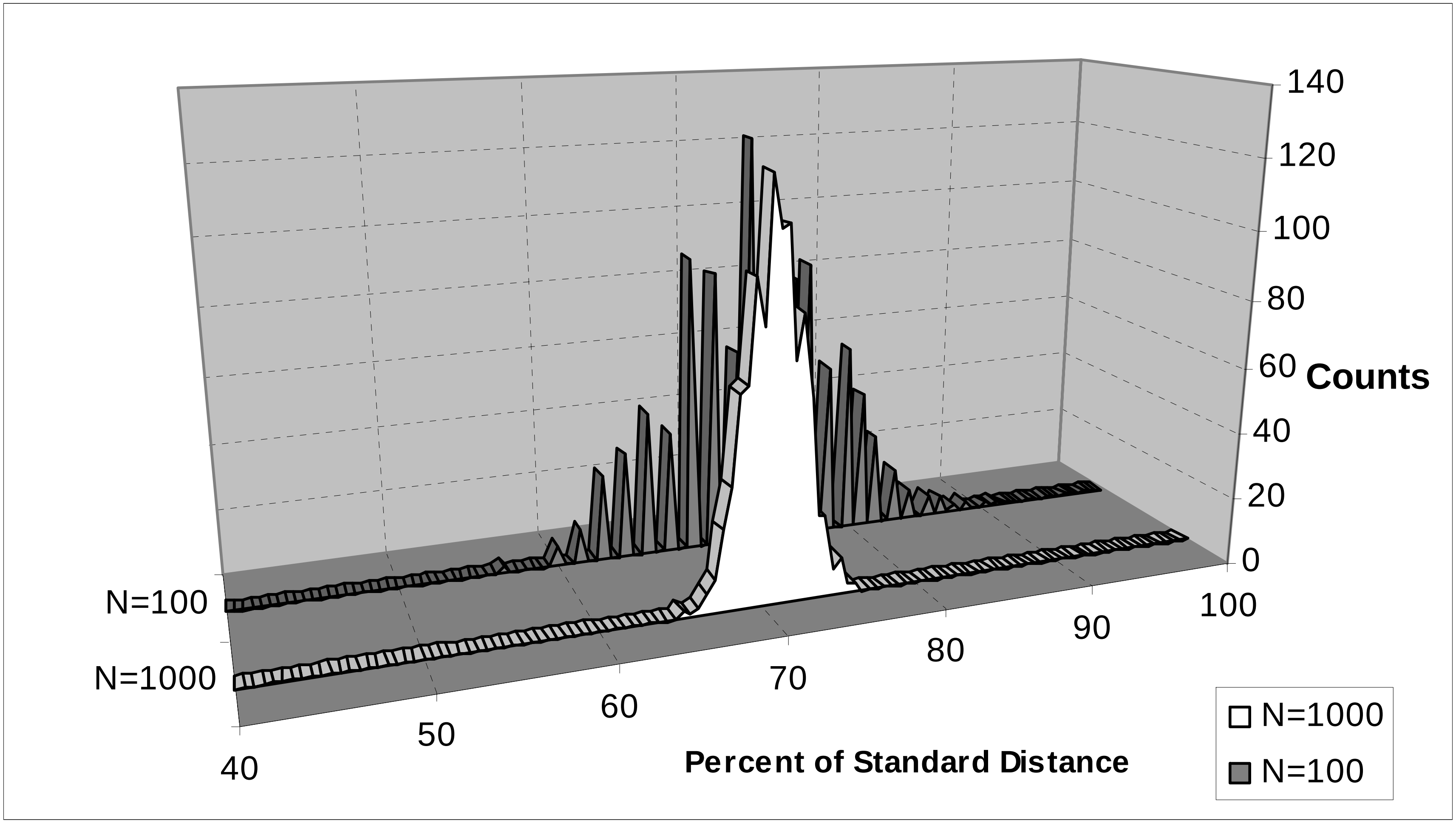


Answers are 75% same from multiple trials of same S_i!!

Repeat Multiple Trials for sets S_i => X and sets S_j => Y

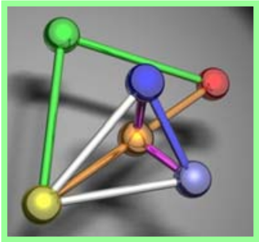
Histogram of COs Survive

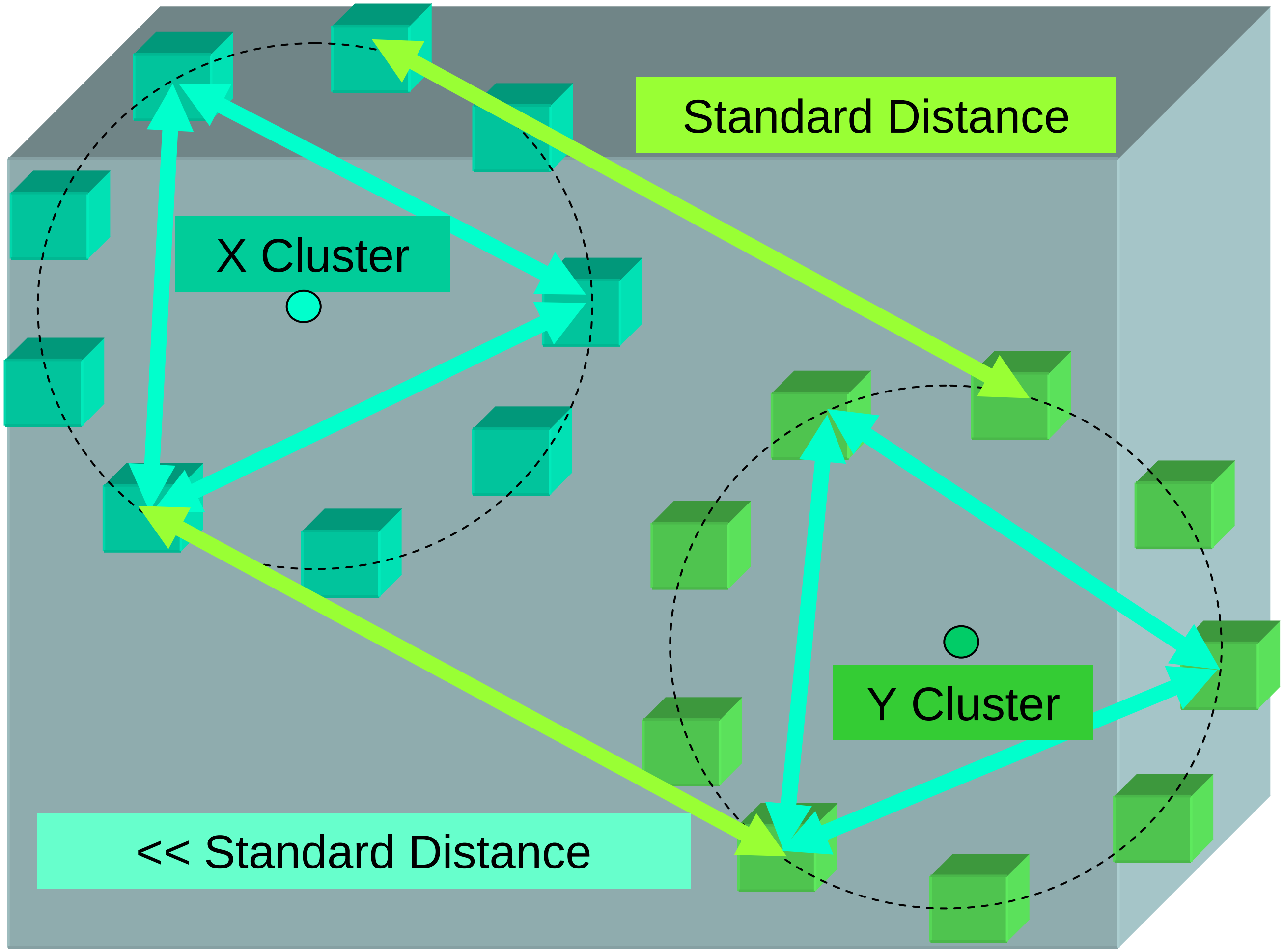




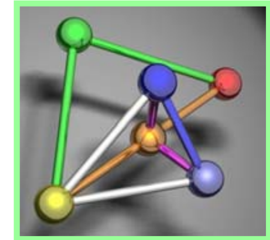
Easily distinguishable because is 70% of standard distance!

Topology of COs Survival





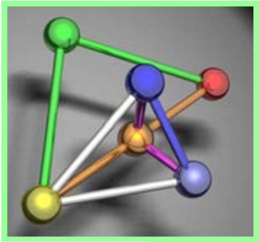
Model Quantum CO process



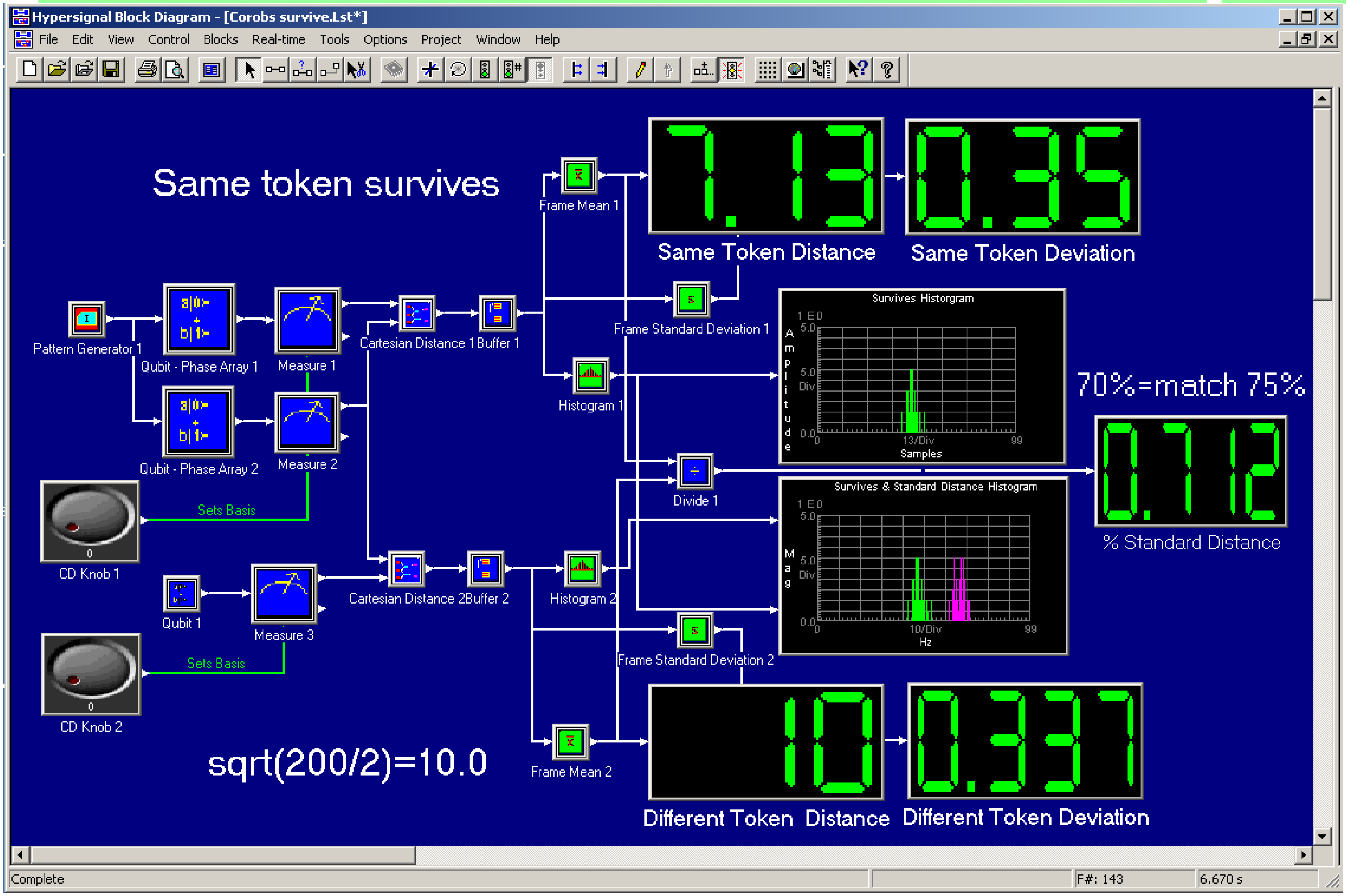
Description of next slide:

- Multiple trials of same CO (top left)
- Multiple trials of random CO (bottom left)
- Make measurements (mid)
- Compare Rand-COs to same CO distances
- Generate histograms (mid)
- Display histograms (right)
- 70% of expected standard distance (right)

Quantum Measurement as COs

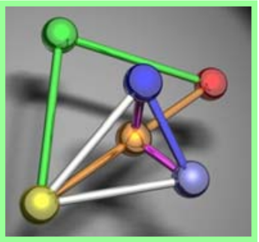


Quantum encoded tokens are distinguishable after measurement

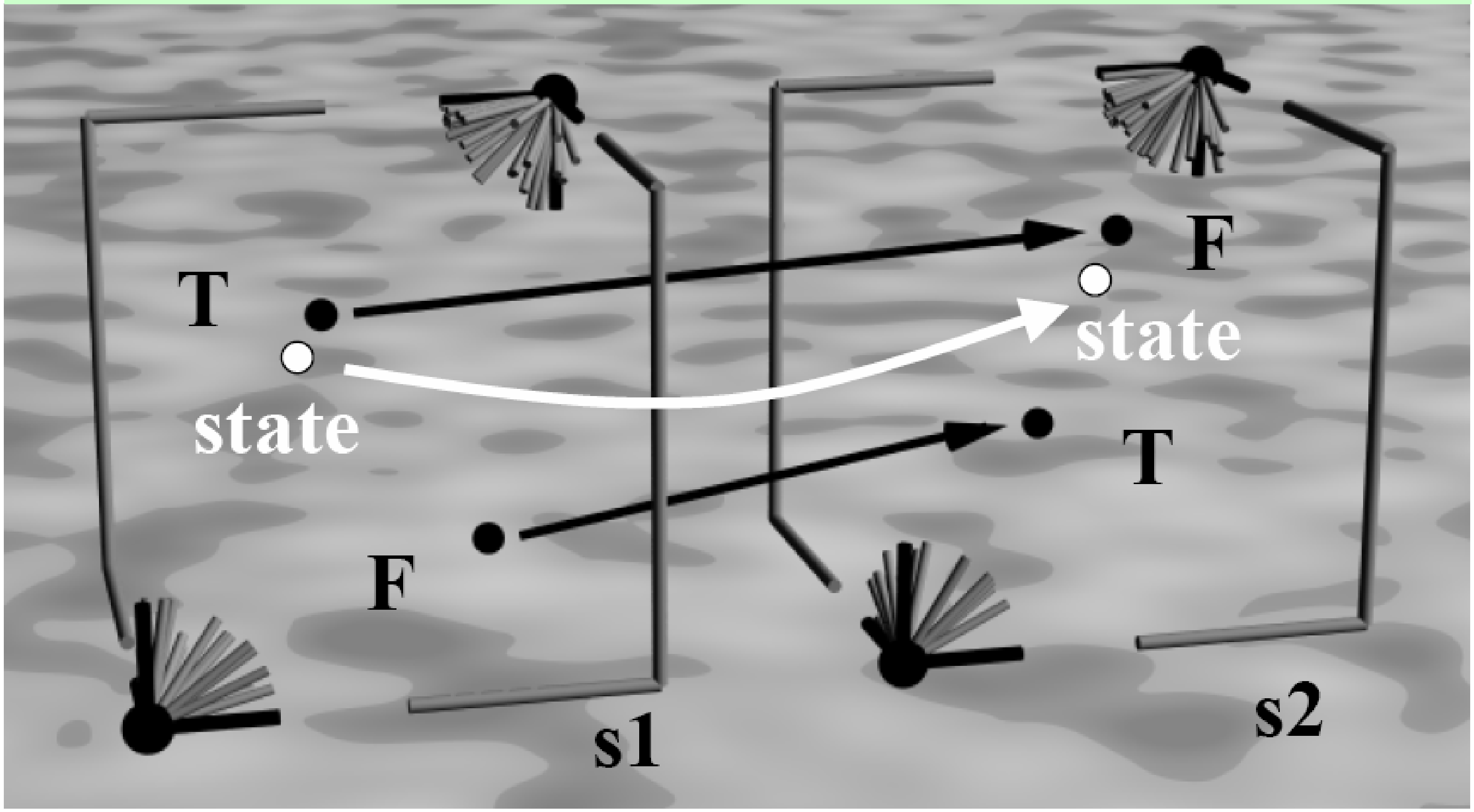


Qu Measurement can be thought of as CO process!

Ensemble State Transformation



CO theory is Boolean and Turing complete plus contains superposition



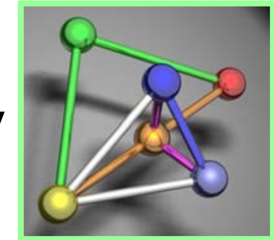
See more examples in our book!

Rules For NOT:
s1.T >: s2.F
s1.F >: s2.T

NOT operator:

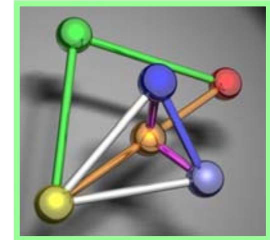
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Quantum Ensemble Summary

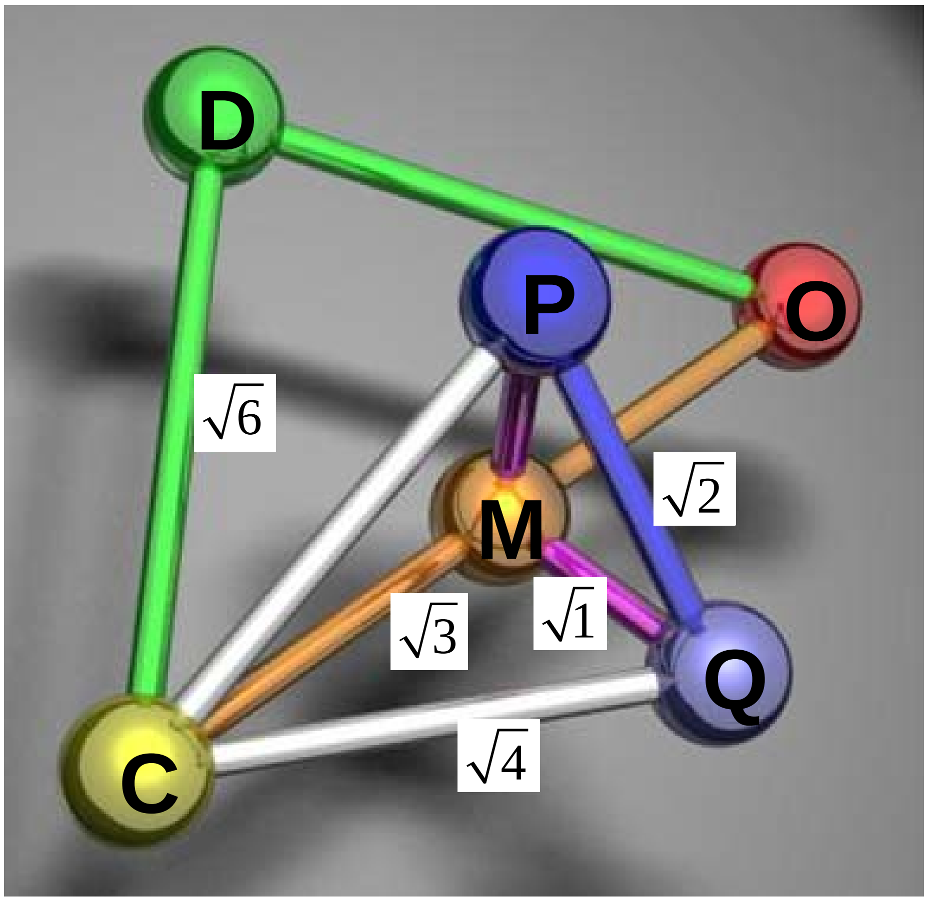


- Invariant Ensemble Metrics
  - Standard distance and standard radius is noise metric
  - Normalized standard deviations → 0 as N → ∞
  - COs are nearly mutually orthogonal
  - Modeling tools support multiple cell types
- Quantum and Correlithm
  - Both depend on probabilities, metrics and info. theory
  - Same standard distance for all Qu ensembles
  - Superposition appears in both domains
- Quantum COs survive measurement
  - Phase invariance gives rise to statistical stability
  - QuMeasurement cast as CO noise injection process

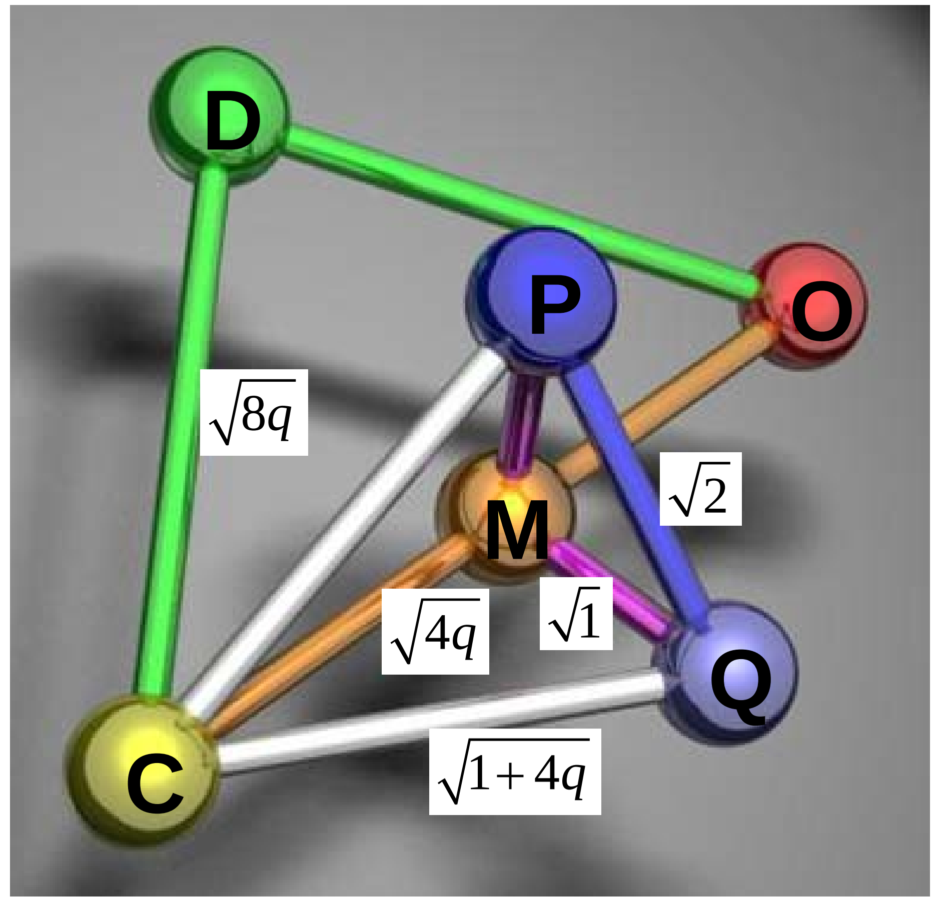
Quantum and Correlithms



Unit N-cube Topology



Quantum Ensemble Topology



Normalized Distance Metrics

Sponsors

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